Chapter 5

# The Laws of Motion

▲ *A small tugboat exerts a force on a large ship, causing it to move. How can such a small boat move such a large object? (Steve Raymer/CORBIS)*

## CHAPTER OUTLINE

In Chapters 2 and 4, we described motion in terms of position, velocity, and acceleration without considering what might cause that motion. Now we consider the cause—what might cause one object to remain at rest and another object to accelerate? The two main factors we need to consider are the forces acting on an object and the mass of the object. We discuss the three basic laws of motion, which deal with forces and masses and were formulated more than three centuries ago by Isaac Newton. Once we understand these laws, we can answer such questions as "What mechanism changes motion?" and "Why do some objects accelerate more than others?"

## 5.1 The Concept of Force

Everyone has a basic understanding of the concept of force from everyday experience. When you push your empty dinner plate away, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word *force* is associated with muscular activity and some change in the velocity of an object. Forces do not always cause motion, however. For example, as you sit reading this book, a gravitational force acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a large boulder and not be able to move it.

What force (if any) causes the Moon to orbit the Earth? Newton answered this and related questions by stating that forces are what cause any change in the velocity of an object. The Moon's velocity is not constant because it moves in a nearly circular orbit around the Earth. We now know that this change in velocity is caused by the gravitational force exerted by the Earth on the Moon. Because only a force can cause a change in velocity, we can think of force as *that which causes an object to accelerate.* In this chapter, we are concerned with the relationship between the force exerted on an object and the acceleration of that object.

**An object accelerates due to an external force**

What happens when several forces act simultaneously on an object? In this case, the object accelerates only if the net force acting on it is not equal to zero. The **net force** acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the *total force*, the *resultant force*, or the *unbalanced force.*) **If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.** That is, if the net force acting on the object is zero, the object either remains at rest or continues to move with constant velocity. When the velocity of an object is constant (including when the object is at rest), the object is said to be in **equilibrium.**

**Definition of equilibrium**

When a coiled spring is pulled, as in Figure 5.1a, the spring stretches. When a stationary cart is pulled sufficiently hard that friction is overcome, as in Figure 5.1b, the cart moves. When a football is kicked, as in Figure 5.1c, it is both deformed and set in motion. These situations are all examples of a class of forces called *contact forces.* That is, they involve physical contact between two objects. Other examples of contact forces are the force exerted by gas molecules on the walls of a container and the force exerted by your feet on the floor.

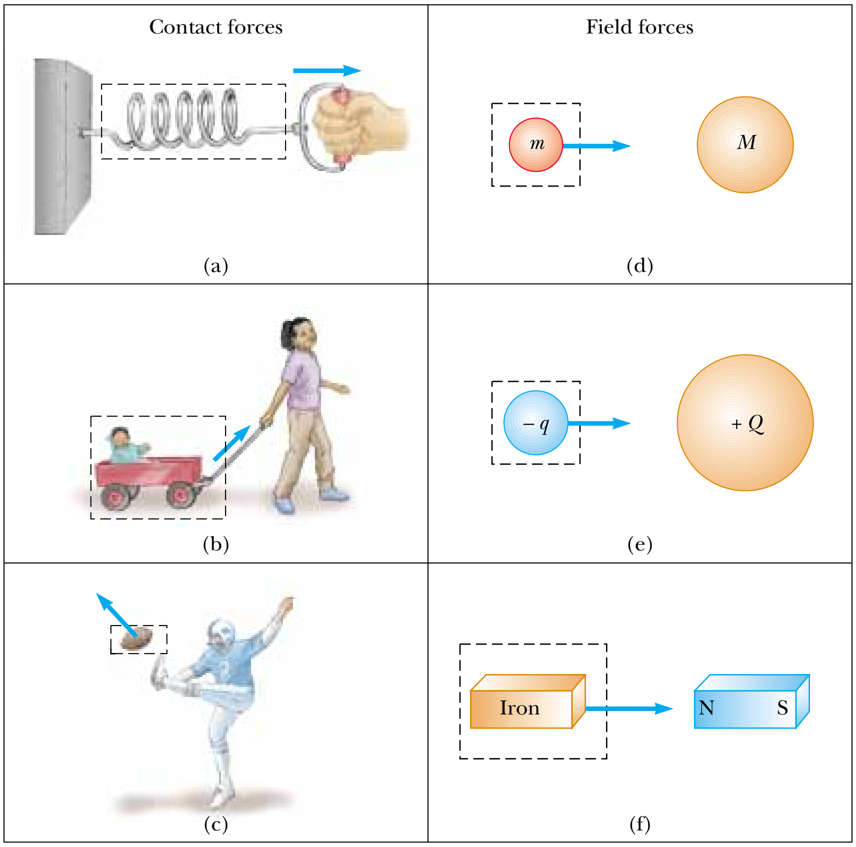



**Figure 5.1** Some examples of applied forces. In each case a force is exerted on the object within the boxed area. Some agent in the environment external to the boxed area exerts a force on the object.

Another class of forces, known as *field forces*, do not involve physical contact between two objects but instead act through empty space. The gravitational force of attraction between two objects, illustrated in Figure 5.1d, is an example of this class of force. This gravitational force keeps objects bound to the Earth and the planets in orbit around the Sun. Another common example of a field force is the electric force that one electric charge exerts on another (Fig. 5.1e). These charges might be those of the electron and proton that form a hydrogen atom. A third example of a field force is the force a bar magnet exerts on a piece of iron (Fig. 5.1f).

The distinction between contact forces and field forces is not as sharp as you may have been led to believe by the previous discussion. When examined at the atomic level, all the forces we classify as contact forces turn out to be caused by electric (field) forces of the type illustrated in Figure 5.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. The only known *fundamental* forces in nature are all field forces: (1) *gravitational forces* between objects, (2) *electromagnetic forces* between electric charges, (3) *nuclear forces* between subatomic particles, and (4) *weak forces* that arise in certain radioactive decay processes. In classical physics, we are concerned only with gravitational and electromagnetic forces.

## Measuring the Strength of a Force

It is convenient to use the deformation of a spring to measure force. Suppose we apply a vertical force to a spring scale that has a fixed upper end, as shown in Figure 5.2a. The spring elongates when the force is applied, and a pointer on the scale reads the value of the applied force. We can calibrate the spring by defining a reference force $\mathbf{F}_1$ as the force that produces a pointer reading of 1.00 cm. (Because force is a vector

**Isaac Newton,**
**English physicist and mathematician (1642–1727)**

Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today. *(Giraudon/Art Resource)*

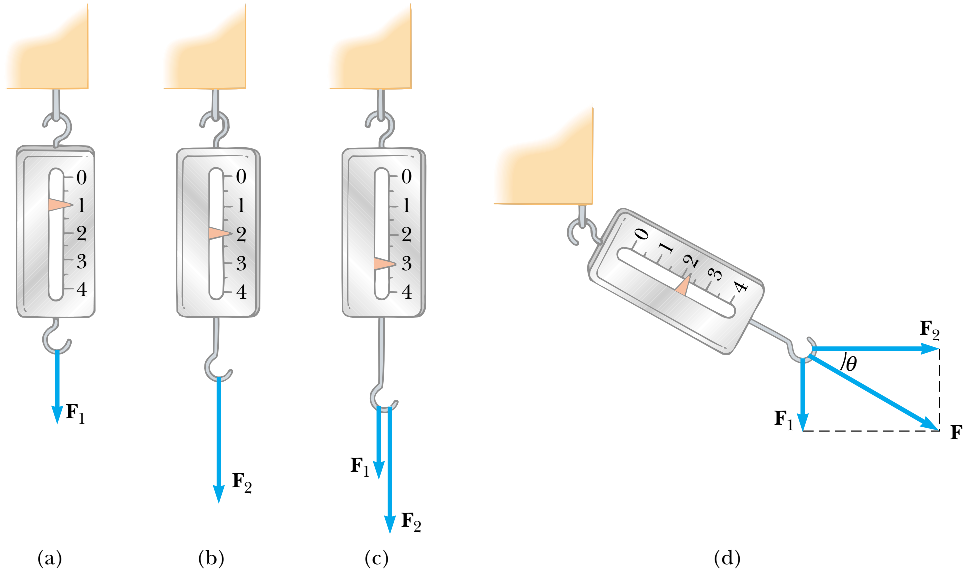



**Figure 5.2** The vector nature of a force is tested with a spring scale. (a) A downward force $\mathbf{F}_1$ elongates the spring 1.00 cm. (b) A downward force $\mathbf{F}_2$ elongates the spring 2.00 cm. (c) When $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied simultaneously, the spring elongates by 3.00 cm. (d) When $\mathbf{F}_1$ is downward and $\mathbf{F}_2$ is horizontal, the combination of the two forces elongates the spring $\sqrt{(1.00\text{ cm})^2 + (2.00\text{ cm})^2} = 2.24$ cm.

quantity, we use the bold-faced symbol **F**.) If we now apply a different downward force $\mathbf{F}_2$ whose magnitude is twice that of the reference force $\mathbf{F}_1$, as seen in Figure 5.2b, the pointer moves to 2.00 cm. Figure 5.2c shows that the combined effect of the two collinear forces is the sum of the effects of the individual forces.

Now suppose the two forces are applied simultaneously with $\mathbf{F}_1$ downward and $\mathbf{F}_2$ horizontal, as illustrated in Figure 5.2d. In this case, the pointer reads $\sqrt{5.00\text{ cm}^2} = 2.24$ cm. The single force **F** that would produce this same reading is the sum of the two vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, as described in Figure 5.2d. That is, $|\mathbf{F}| = \sqrt{F_1^2 + F_2^2} = 2.24$ units, and its direction is $\theta = \tan^{-1}(-0.500) = -26.6°$. **Because forces have been experimentally verified to behave as vectors, you must use the rules of vector addition to obtain the net force on an object.**

## 5.2 Newton's First Law and Inertial Frames

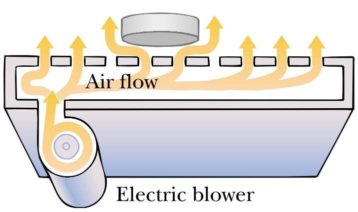



**Figure 5.3** On an air hockey table, air blown through holes in the surface allow the puck to move almost without friction. If the table is not accelerating, a puck placed on the table will remain at rest.

We begin our study of forces by imagining some situations. Imagine placing a puck on a perfectly level air hockey table (Fig. 5.3). You expect that it will remain where it is placed. Now imagine your air hockey table is located on a train moving with constant velocity. If the puck is placed on the table, the puck again remains where it is placed. If the train were to accelerate, however, the puck would start moving along the table, just as a set of papers on your dashboard falls onto the front seat of your car when you step on the gas.

As we saw in Section 4.6, a moving object can be observed from any number of reference frames. **Newton's first law of motion,** sometimes called the *law of inertia,* defines a special set of reference frames called *inertial frames.* This law can be stated as follows:

**Newton's first law**

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

Such a reference frame is called an **inertial frame of reference**. When the puck is on the air hockey table located on the ground, you are observing it from an inertial reference frame—there are no horizontal interactions of the puck with any other objects and you observe it to have zero acceleration in that direction. When you are on the train moving at constant velocity, you are also observing the puck from an inertial reference frame. **Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame**. When the train accelerates, however, you are observing the puck from a **noninertial reference frame** because you and the train are accelerating relative to the inertial reference frame of the surface of the Earth. While the puck appears to be accelerating according to your observations, we can identify a reference frame in which the puck has zero acceleration. For example, an observer standing outside the train on the ground sees the puck moving with the same velocity as the train had before it started to accelerate (because there is almost no friction to "tie" the puck and the train together). Thus, Newton's first law is still satisfied even though your observations say otherwise.

**Inertial frame of reference**

A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame, and for our purposes we can consider the Earth as being such a frame. The Earth is not really an inertial frame because of its orbital motion around the Sun and its rotational motion about its own axis, both of which result in centripetal accelerations. However, these accelerations are small compared with $g$ and can often be neglected. For this reason, we assume that the Earth is an inertial frame, as is any other frame attached to it.

Let us assume that we are observing an object from an inertial reference frame. (We will return to observations made in noninertial reference frames in Section 6.3.) Before about 1600, scientists believed that the natural state of matter was the state of rest. Observations showed that moving objects eventually stopped moving. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments and concluded that it is not the nature of an object to stop once set in motion: rather, it is its nature to *resist changes in its motion.* In his words, "Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed." For example, a spacecraft drifting through empty space with its engine turned off will keep moving forever—it would *not* seek a "natural state" of rest.

**PITFALL PREVENTION**

**5.1 Newton's First Law**

Newton's first law does *not* say what happens for an object with *zero net force,* that is, multiple forces that cancel; it says what happens *in the absence of a force.* This is a subtle but important difference that allows us to define force as that which causes a change in the motion. The description of an object under the effect of forces that balance is covered by Newton's second law.

Given our assumption of observations made from inertial reference frames, we can pose a more practical statement of Newton's first law of motion:

**Another statement of Newton's first law**

> In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

In simpler terms, we can say that **when no force acts on an object, the acceleration of the object is zero.** If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any *isolated object* (one that does not interact with its environment) is either at rest or moving with constant velocity. The tendency of an object to resist any attempt to change its velocity is called **inertia.**

**Quick Quiz 5.1** Which of the following statements is most correct? (a) It is possible for an object to have motion in the absence of forces on the object. (b) It is possible to have forces on an object in the absence of motion of the object. (c) Neither (a) nor (b) is correct. (d) Both (a) and (b) are correct.

## 5.3 Mass

Imagine playing catch with either a basketball or a bowling ball. Which ball is more likely to keep moving when you try to catch it? Which ball has the greater tendency to remain motionless when you try to throw it? The bowling ball is more resistant to changes in its velocity than the basketball—how can we quantify this concept?

**Definition of mass**

**Mass** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity, and as we learned in Section 1.1, the SI unit of mass is the kilogram. The greater the mass of an object, the less that object accelerates under the action of a given applied force.

To describe mass quantitatively, we begin by experimentally comparing the accelerations a given force produces on different objects. Suppose a force acting on an object of mass $m_1$ produces an acceleration $\mathbf{a}_1$, and the *same force* acting on an object of mass $m_2$ produces an acceleration $\mathbf{a}_2$. The ratio of the two masses is defined as the *inverse* ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1} \tag{5.1}$$

For example, if a given force acting on a 3-kg object produces an acceleration of 4 $m/s^2$, the same force applied to a 6-kg object produces an acceleration of 2 $m/s^2$. If one object has a known mass, the mass of the other object can be obtained from acceleration measurements.

**Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.** Also, **mass is a scalar quantity** and thus obeys the rules of ordinary arithmetic. That is, several masses can be combined in simple numerical fashion. For example, if you combine a 3-kg mass with a 5-kg mass, the total mass is 8 kg. We can verify this result experimentally by comparing the accelerations that a known force gives to several objects separately with the acceleration that the same force gives to the same objects combined as one unit.

**Mass and weight are different quantities**

Mass should not be confused with weight. **Mass and weight are two different quantities.** The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location (see Section 5.5). For example, a person who weighs 180 lb on the Earth weighs only about 30 lb on the Moon. On the other hand, the mass of an object is the same everywhere: an object having a mass of 2 kg on the Earth also has a mass of 2 kg on the Moon.

## 5.4 Newton's Second Law

Newton's first law explains what happens to an object when no forces act on it. It either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a nonzero resultant force acting on it.

Imagine performing an experiment in which you push a block of ice across a frictionless horizontal surface. When you exert some horizontal force $\mathbf{F}$ on the block, it moves with some acceleration $\mathbf{a}$. If you apply a force twice as great, you find that the acceleration of the block doubles. If you increase the applied force to $3\mathbf{F}$, the acceleration triples, and so on. From such observations, we conclude that **the acceleration of an object is directly proportional to the force acting on it.**

The acceleration of an object also depends on its mass, as stated in the preceding section. We can understand this by considering the following experiment. If you apply a force $\mathbf{F}$ to a block of ice on a frictionless surface, the block undergoes some acceleration $\mathbf{a}$. If the mass of the block is doubled, the same applied force produces an acceleration $\mathbf{a}/2$. If the mass is tripled, the same applied force produces an acceleration $\mathbf{a}/3$, and so on. According to this observation, we conclude that **the magnitude of the acceleration of an object is inversely proportional to its mass.**

These observations are summarized in **Newton's second law:**

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Thus, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:[1]

$$\sum \mathbf{F} = m\mathbf{a} \tag{5.2}$$

Newton's second law

In both the textual and mathematical statements of Newton's second law above, we have indicated that the acceleration is due to the *net force* $\Sigma\mathbf{F}$ acting on an object. The **net force** on an object is the vector sum of all forces acting on the object. In solving a problem using Newton's second law, it is imperative to determine the correct net force on an object. There may be many forces acting on an object, but there is only one acceleration.

Note that Equation 5.2 is a vector expression and hence is equivalent to three component equations:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{5.3}$$

Newton's second law–component form

**Quick Quiz 5.2** An object experiences no acceleration. Which of the following *cannot* be true for the object? (a) A single force acts on the object. (b) No forces act on the object. (c) Forces act on the object, but the forces cancel.

**Quick Quiz 5.3** An object experiences a net force and exhibits an acceleration in response. Which of the following statements is *always* true? (a) The object moves in the direction of the force. (b) The acceleration is in the same direction as the velocity. (c) The acceleration is in the same direction as the force. (d) The velocity of the object increases.

**Quick Quiz 5.4** You push an object, initially at rest, across a frictionless floor with a constant force for a time interval $\Delta t$, resulting in a final speed of $v$ for the object. You repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed $v$? (a) $4\,\Delta t$ (b) $2\,\Delta t$ (c) $\Delta t$ (d) $\Delta t/2$ (e) $\Delta t/4$.

## Unit of Force

The SI unit of force is the **newton**, which is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s$^2$. From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

$$1\text{ N} \equiv 1\text{ kg}\cdot\text{m/s}^2 \tag{5.4}$$

Definition of the newton

[1] Equation 5.2 is valid only when the speed of the object is much less than the speed of light. We treat the relativistic situation in Chapter 39.

**PITFALL PREVENTION**

**5.2 Force is the Cause of Changes in Motion**

Force *does not* cause motion. We can have motion in the absence of forces, as described in Newton's first law. Force is the cause of *changes* in motion, as measured by acceleration.

**PITFALL PREVENTION**

**5.3 *m*a is Not a Force**

Equation 5.2 does *not* say that the product $m\mathbf{a}$ is a force. All forces on an object are added vectorially to generate the net force on the left side of the equation. This net force is then equated to the product of the mass of the object and the acceleration that results from the net force. Do *not* include an "$m\mathbf{a}$ force" in your analysis of the forces on an object.

**Table 5.1**

**Units of Mass, Acceleration, and Force**[a]

| System of Units | Mass | Acceleration | Force |
|---|---|---|---|
| SI | kg | $m/s^2$ | $N = kg \cdot m/s^2$ |
| U.S. customary | slug | $ft/s^2$ | $lb = slug \cdot ft/s^2$ |

[a] 1 N = 0.225 lb.

In the U.S. customary system, the unit of force is the **pound**, which is defined as the force that, when acting on a 1-slug mass,[2] produces an acceleration of 1 $ft/s^2$:

$$1\,\text{lb} \equiv 1\,\text{slug} \cdot \text{ft/s}^2 \tag{5.5}$$

A convenient approximation is that $1\,\text{N} \approx \frac{1}{4}$ lb.

The units of mass, acceleration, and force are summarized in Table 5.1.

## Example 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the horizontal, frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\mathbf{F}_1$ has a magnitude of 5.0 N, and the force $\mathbf{F}_2$ has a magnitude of 8.0 N. Determine both the magnitude and the direction of the puck's acceleration.

***Solution*** *Conceptualize* this problem by studying Figure 5.4. Because we can determine a net force and we want an acceleration, we *categorize* this problem as one that may be solved using Newton's second law. To *analyze* the problem, we resolve the force vectors into components. The net force acting on the puck in the $x$ direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$
$$= (5.0\text{ N})(0.940) + (8.0\text{ N})(0.500) = 8.7\text{ N}$$

The net force acting on the puck in the $y$ direction is

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$
$$= (5.0\text{ N})(-0.342) + (8.0\text{ N})(0.866) = 5.2\text{ N}$$

Now we use Newton's second law in component form to find the $x$ and $y$ components of the puck's acceleration:

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\text{ N}}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\text{ N}}{0.30\text{ kg}} = 17\text{ m/s}^2$$

The acceleration has a magnitude of

$$a = \sqrt{(29)^2 + (17)^2}\text{ m/s}^2 = \boxed{34\text{ m/s}^2}$$

and its direction relative to the positive $x$ axis is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{30°}$$

To *finalize* the problem, we can graphically add the vectors in Figure 5.4 to check the reasonableness of our answer. Because the acceleration vector is along the direction of the resultant force, a drawing showing the resultant force helps us check the validity of the answer. (Try it!)

**What If? Suppose three hockey sticks strike the puck simultaneously, with two of them exerting the forces shown in Figure 5.4. The result of the three forces is that the hockey puck shows *no* acceleration. What must be the components of the third force?**

***Answer*** If there is zero acceleration, the net force acting on the puck must be zero. Thus, the three forces must cancel. We have found the components of the combination of the first two forces. The components of the third force must be of equal magnitude and opposite sign in order that all of the components add to zero. Thus, $F_{3x} = -8.7$ N, $F_{3y} = -5.2$ N.

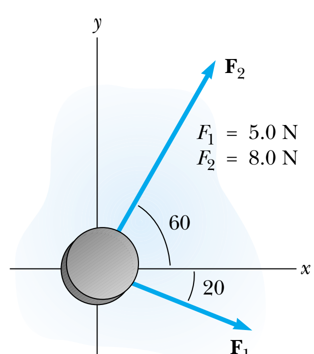



**Figure 5.4** (Example 5.1) A hockey puck moving on a frictionless surface accelerates in the direction of the resultant force $\mathbf{F}_1 + \mathbf{F}_2$.

[2] The *slug* is the unit of mass in the U.S. customary system and is that system's counterpart of the SI unit the *kilogram*. Because most of the calculations in our study of classical mechanics are in SI units, the slug is seldom used in this text.

## 5.5 The Gravitational Force and Weight

We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** $\mathbf{F}_g$. This force is directed toward the center of the Earth,[3] and its magnitude is called the **weight** of the object.

We saw in Section 2.6 that a freely falling object experiences an acceleration $\mathbf{g}$ acting toward the center of the Earth. Applying Newton's second law $\Sigma\mathbf{F} = m\mathbf{a}$ to a freely falling object of mass $m$, with $\mathbf{a} = \mathbf{g}$ and $\Sigma\mathbf{F} = \mathbf{F}_g$, we obtain

$$\mathbf{F}_g = m\mathbf{g} \tag{5.6}$$

Thus, the weight of an object, being defined as the magnitude of $\mathbf{F}_g$, is equal to $mg$.

Because it depends on $g$, weight varies with geographic location. Because $g$ decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level. For example, a 1 000-kg palette of bricks used in the construction of the Empire State Building in New York City weighed 9 800 N at street level, but weighed about 1 N less by the time it was lifted from sidewalk level to the top of the building. As another example, suppose a student has a mass of 70.0 kg. The student's weight in a location where $g = 9.80\ \text{m/s}^2$ is $F_g = mg = 686$ N (about 150 lb). At the top of a mountain, however, where $g = 9.77\ \text{m/s}^2$, the student's weight is only 684 N. Therefore, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight!

Because weight is proportional to mass, we can compare the masses of two objects by measuring their weights on a spring scale. At a given location (at which two objects are subject to the same value of $g$), the ratio of the weights of two objects equals the ratio of their masses.

Equation 5.6 quantifies the gravitational force on the object, but notice that this equation does not require the object to be moving. Even for a stationary object, or an object on which several forces act, Equation 5.6 can be used to calculate the magnitude of the gravitational force. This results in a subtle shift in the interpretation of $m$ in the equation. The mass $m$ in Equation 5.6 is playing the role of determining the strength of the gravitational attraction between the object and the Earth. This is a completely different role from that previously described for mass, that of measuring the resistance to changes in motion in response to an external force. Thus, we call $m$ in this type of equation the **gravitational mass.** Despite this quantity being different in behavior from inertial mass, it is one of the experimental conclusions in Newtonian dynamics that gravitational mass and inertial mass have the same value.

**Quick Quiz 5.5** A baseball of mass $m$ is thrown upward with some initial speed. A gravitational force is exerted on the ball (a) at all points in its motion (b) at all points in its motion except at the highest point (c) at no points in its motion.

**Quick Quiz 5.6** Suppose you are talking by interplanetary telephone to your friend, who lives on the Moon. He tells you that he has just won a newton of gold in a contest. Excitedly, you tell him that you entered the Earth version of the same contest and also won a newton of gold! Who is richer? (a) You (b) Your friend (c) You are equally rich.

[3] This statement ignores the fact that the mass distribution of the Earth is not perfectly spherical.

**PITFALL PREVENTION**

**5.4 "Weight of an Object"**

We are familiar with the everyday phrase, the "weight of an object." However, weight is not an inherent property of an object, but rather a measure of the gravitational force between the object and the Earth. Thus, weight is a property of a *system* of items—the object and the Earth.

**PITFALL PREVENTION**

**5.5 Kilogram is Not a Unit of Weight**

You may have seen the "conversion" 1 kg = 2.2 lb. Despite popular statements of weights expressed in kilograms, the kilogram is not a unit of *weight*, it is a unit of *mass*. The conversion statement is not an equality; it is an *equivalence* that is only valid on the surface of the Earth.

Courtesy of NASA

The life-support unit strapped to the back of astronaut Edwin Aldrin weighed 300 lb on the Earth. During his training, a 50-lb mock-up was used. Although this effectively simulated the reduced weight the unit would have on the Moon, it did not correctly mimic the unchanging mass. It was just as difficult to accelerate the unit (perhaps by jumping or twisting suddenly) on the Moon as on the Earth.

**Conceptual Example 5.2 How Much Do You Weigh in an Elevator?**

You have most likely had the experience of standing in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force having a magnitude that is greater than your weight. Thus, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are* you heavier?

***Solution*** No, your weight is unchanged. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force that you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

## 5.6 Newton's Third Law

If you press against a corner of this textbook with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin is a little larger. This simple experiment illustrates a general principle of critical importance known as **Newton's third law:**

**Newton's third law**

If two objects interact, the force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \tag{5.7}$$

When it is important to designate forces as interactions between two objects, we will use this subscript notation, where $\mathbf{F}_{ab}$ means "the force exerted *by* a *on* b." The third law, which is illustrated in Figure 5.5a, is equivalent to stating that **forces always occur in pairs,** or that **a single isolated force cannot exist.** The force that object 1 exerts on object 2 may be called the *action force* and the force of object 2 on object 1 the *reaction force*. In reality, either force can be labeled the action or reaction force. **The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects and must be of the same type.** For example, the force acting on a freely falling projectile is the gravitational force exerted by the Earth on the projectile $\mathbf{F}_g = \mathbf{F}_{Ep}$ (E = Earth, p = projectile), and the magnitude of this force is $mg$. The reaction to this force is the gravitational force exerted by the projectile on the Earth $\mathbf{F}_{pE} = -\mathbf{F}_{Ep}$. The reaction force $\mathbf{F}_{pE}$ must accelerate the Earth toward the projectile just as the action force $\mathbf{F}_{Ep}$ accelerates the projectile toward the Earth. However, because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.

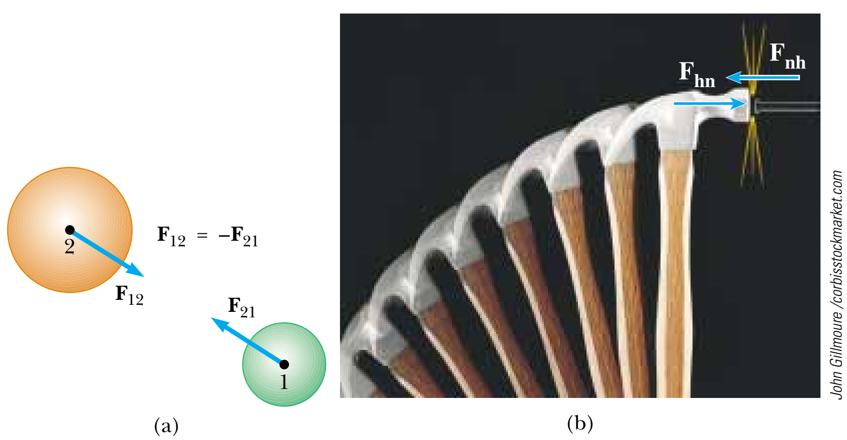
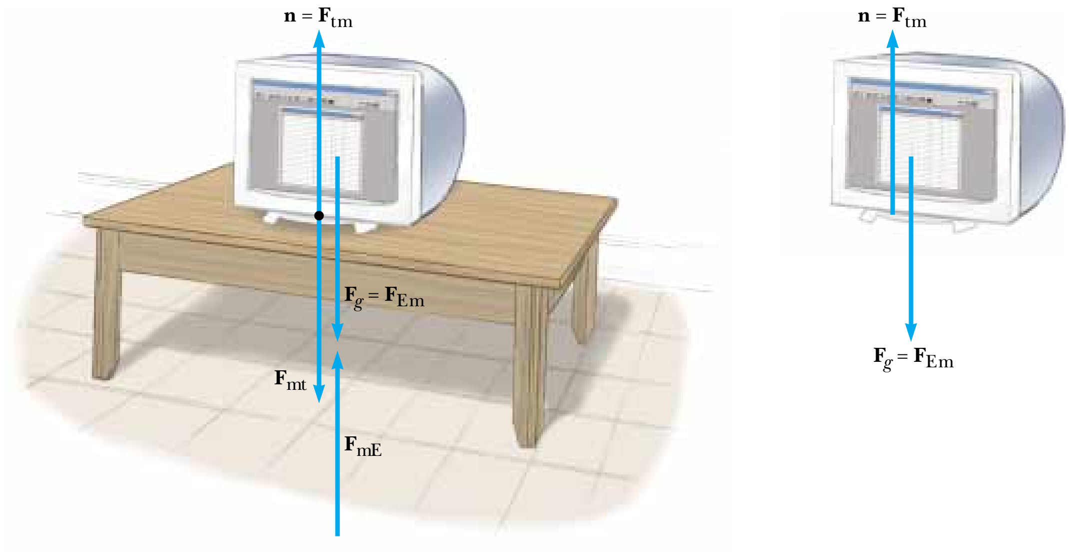



**Figure 5.5** Newton's third law. (a) The force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1. (b) The force $\mathbf{F}_{hn}$ exerted by the hammer on the nail is equal in magnitude and opposite to the force $\mathbf{F}_{nh}$ exerted by the nail on the hammer.




(a) (b)

**Figure 5.6** (a) When a computer monitor is at rest on a table, the forces acting on the monitor are the normal force $\mathbf{n}$ and the gravitational force $\mathbf{F}_g$. The reaction to $\mathbf{n}$ is the force $\mathbf{F}_{mt}$ exerted by the monitor on the table. The reaction to $\mathbf{F}_g$ is the force $\mathbf{F}_{mE}$ exerted by the monitor on the Earth. (b) The free-body diagram for the monitor.

Another example of Newton's third law is shown in Figure 5.5b. The force $\mathbf{F}_{hn}$ exerted by the hammer on the nail (the action) is equal in magnitude and opposite the force $\mathbf{F}_{nh}$ exerted by the nail on the hammer (the reaction). This latter force stops the forward motion of the hammer when it strikes the nail.

You experience the third law directly if you slam your fist against a wall or kick a football with your bare foot. You can feel the force back on your fist or your foot. You should be able to identify the action and reaction forces in these cases.

The Earth exerts a gravitational force $\mathbf{F}_g$ on any object. If the object is a computer monitor at rest on a table, as in Figure 5.6a, the reaction force to $\mathbf{F}_g = \mathbf{F}_{Em}$ is the force exerted by the monitor on the Earth $\mathbf{F}_{mE} = -\mathbf{F}_{Em}$. The monitor does not accelerate because it is held up by the table. The table exerts on the monitor an upward force $\mathbf{n} = \mathbf{F}_{tm}$, called the **normal force**.[4] This is the force that prevents the monitor from falling through the table; it can have any value needed, up to the point of breaking the table. From Newton's second law, we see that, because the monitor has zero acceleration, it follows that $\Sigma\mathbf{F} = \mathbf{n} - m\mathbf{g} = 0$, or $n = mg$. The normal force balances the gravitational force on the monitor, so that the net force on the monitor is zero. The reaction to $\mathbf{n}$ is the force exerted by the monitor downward on the table, $\mathbf{F}_{mt} = -\mathbf{F}_{tm} = -\mathbf{n}$.

Definition of normal force

Note that the forces acting on the monitor are $\mathbf{F}_g$ and $\mathbf{n}$, as shown in Figure 5.6b. The two reaction forces $\mathbf{F}_{mE}$ and $\mathbf{F}_{mt}$ are exerted on objects other than the monitor. Remember, **the two forces in an action–reaction pair always act on two different objects.**

Figure 5.6 illustrates an extremely important step in solving problems involving forces. Figure 5.6a shows many of the forces in the situation—those acting on the monitor, one acting on the table, and one acting on the Earth. Figure 5.6b, by contrast, shows only the forces acting on *one object*, the monitor. This is a critical drawing called a **free-body diagram**. When analyzing an object subject to forces, we are interested in the net force acting on one object, which we will model as a particle. Thus, a free-body diagram helps us to isolate only those forces on the object and eliminate the other forces from our analysis. The free-body diagram can be simplified further by representing the object (such as the monitor) as a particle, by simply drawing a dot.

### PITFALL PREVENTION

#### 5.6 *n* Does Not Always Equal *mg*

In the situation shown in Figure 5.6 and in many others, we find that $n = mg$ (the normal force has the same magnitude as the gravitational force). However, this is *not* generally true. If an object is on an incline, if there are applied forces with vertical components, or if there is a vertical acceleration of the system, then $n \neq mg$. *Always* apply Newton's second law to find the relationship between $n$ and $mg$.

### PITFALL PREVENTION

#### 5.7 Newton's Third Law

This is such an important and often misunderstood concept that it will be repeated here in a Pitfall Prevention. Newton's third law action and reaction forces act on *different* objects. Two forces acting on the same object, even if they are equal in magnitude and opposite in direction, *cannot* be an action–reaction pair.

[4] *Normal* in this context means *perpendicular.*

**PITFALL PREVENTION**

**5.8 Free-body Diagrams**

The *most important* step in solving a problem using Newton's laws is to draw a proper sketch—the free-body diagram. Be sure to draw only those forces that act on the object that you are isolating. Be sure to draw *all* forces acting on the object, including any field forces, such as the gravitational force.

**Quick Quiz 5.7** If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude? (a) the fly (b) the bus (c) the same force is experienced by both.

**Quick Quiz 5.8** If a fly collides with the windshield of a fast-moving bus, which object experiences the greater acceleration: (a) the fly (b) the bus (c) the same acceleration is experienced by both.

**Quick Quiz 5.9** Which of the following is the reaction force to the gravitational force acting on your body as you sit in your desk chair? (a) The normal force exerted by the chair (b) The force you exert downward on the seat of the chair (c) Neither of these forces.

**Quick Quiz 5.10** In a free-body diagram for a single object, you draw (a) the forces acting on the object and the forces the object exerts on other objects, or (b) only the forces acting on the object.

**Conceptual Example 5.3 You Push Me and I'll Push You**

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

**(A)** Who moves away with the higher speed?

***Solution*** This situation is similar to what we saw in Quick Quizzes 5.7 and 5.8. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are an action–reaction pair, and so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.) Therefore, the boy, having the smaller mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

**(B)** Who moves farther while their hands are in contact?

***Solution*** Because the boy has the greater acceleration and, therefore, the greater average velocity, he moves farther during the time interval in which the hands are in contact.

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Rock climbers at rest are in equilibrium and depend on the tension forces in ropes for their safety.

## 5.7 Some Applications of Newton's Laws

In this section we apply Newton's laws to objects that are either in equilibrium ($\mathbf{a} = 0$) or accelerating along a straight line under the action of constant external forces. Remember that **when we apply Newton's laws to an object, we are interested only in external forces that act on the object.** We assume that the objects can be modeled as particles so that we need not worry about rotational motion. For now, we also neglect the effects of friction in those problems involving motion; this is equivalent to stating that the surfaces are *frictionless.* (We will incorporate the friction force in problems in Section 5.8.)

We usually neglect the mass of any ropes, strings, or cables involved. In this approximation, the magnitude of the force exerted at any point along a rope is the same at all points along the rope. In problem statements, the synonymous terms *light* and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problems. When a rope attached to an object is pulling on the object, the rope exerts a force $\mathbf{T}$ on the object, and the magnitude $T$ of that force is called the **tension** in the rope. Because it is the magnitude of a vector quantity, tension is a scalar quantity.

## Objects in Equilibrium

If the acceleration of an object that can be modeled as a particle is zero, the particle is in **equilibrium**. Consider a lamp suspended from a light chain fastened to the ceiling, as in Figure 5.7a. The free-body diagram for the lamp (Figure 5.7b) shows that the forces acting on the lamp are the downward gravitational force $\mathbf{F}_g$ and the upward force $\mathbf{T}$ exerted by the chain. If we apply the second law to the lamp, noting that $\mathbf{a} = 0$, we see that because there are no forces in the $x$ direction, $\Sigma F_x = 0$ provides no helpful information. The condition $\Sigma F_y = ma_y = 0$ gives

$$\sum F_y = T - F_g = 0 \qquad \text{or} \qquad T = F_g$$

Again, note that $\mathbf{T}$ and $\mathbf{F}_g$ are *not* an action–reaction pair because they act on the same object—the lamp. The reaction force to $\mathbf{T}$ is $\mathbf{T}'$, the downward force exerted by the lamp on the chain, as shown in Figure 5.7c. The ceiling exerts on the chain a force $\mathbf{T}''$ that is equal in magnitude to the magnitude of $\mathbf{T}'$ and points in the opposite direction.

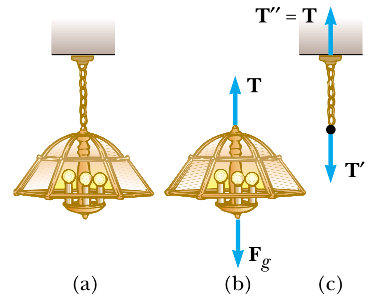



**Figure 5.7** (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the gravitational force $\mathbf{F}_g$ and the force $\mathbf{T}$ exerted by the chain. (c) The forces acting on the chain are the force $\mathbf{T}'$ exerted by the lamp and the force $\mathbf{T}''$ exerted by the ceiling.

## Objects Experiencing a Net Force

If an object that can be modeled as a particle experiences an acceleration, then there must be a nonzero net force acting on the object. Consider a crate being pulled to the right on a frictionless, horizontal surface, as in Figure 5.8a. Suppose you are asked to find the acceleration of the crate and the force the floor exerts on it. First, note that the horizontal force $\mathbf{T}$ being applied to the crate acts through the rope. The magnitude of $\mathbf{T}$ is equal to the tension in the rope. The forces acting on the crate are illustrated in the free-body diagram in Figure 5.8b. In addition to the force $\mathbf{T}$, the free-body diagram for the crate includes the gravitational force $\mathbf{F}_g$ and the normal force $\mathbf{n}$ exerted by the floor on the crate.

We can now apply Newton's second law in component form to the crate. The only force acting in the $x$ direction is $\mathbf{T}$. Applying $\Sigma F_x = ma_x$ to the horizontal motion gives

$$\sum F_x = T = ma_x \qquad \text{or} \qquad a_x = \frac{T}{m}$$

No acceleration occurs in the $y$ direction. Applying $\Sigma F_y = ma_y$ with $a_y = 0$ yields

$$n + (-F_g) = 0 \qquad \text{or} \qquad n = F_g$$

That is, the normal force has the same magnitude as the gravitational force but acts in the opposite direction.

If $\mathbf{T}$ is a constant force, then the acceleration $a_x = T/m$ also is constant. Hence, the constant-acceleration equations of kinematics from Chapter 2 can be used to obtain the crate's position $x$ and velocity $v_x$ as functions of time. Because $a_x = T/m =$ constant, Equations 2.9 and 2.12 can be written as

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}\left(\frac{T}{m}\right)t^2$$

In the situation just described, the magnitude of the normal force $\mathbf{n}$ is equal to the magnitude of $\mathbf{F}_g$, but this is not always the case. For example, suppose a book is lying on a table and you push down on the book with a force $\mathbf{F}$, as in Figure 5.9. Because the book is at rest and therefore not accelerating, $\Sigma F_y = 0$, which gives $n - F_g - F = 0$, or $n = F_g + F$. In this situation, the normal force is *greater* than the force of gravity. Other examples in which $n \neq F_g$ are presented later.

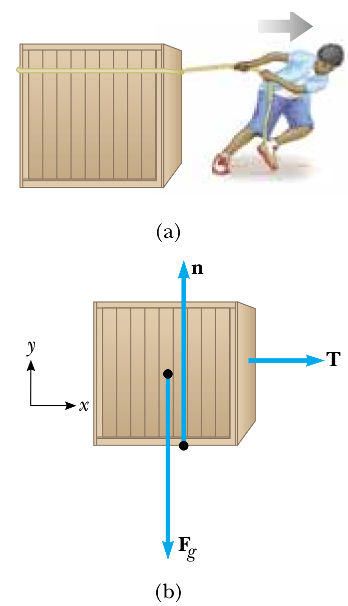



**Figure 5.8** (a) A crate being pulled to the right on a frictionless surface. (b) The free-body diagram representing the external forces acting on the crate.

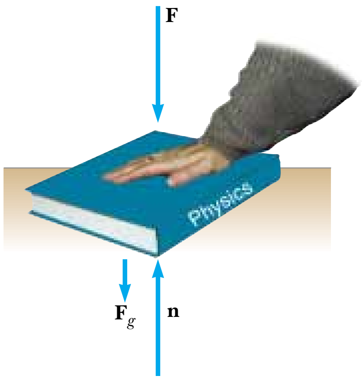



**Figure 5.9** When one object pushes downward on another object with a force **F**, the normal force **n** is greater than the gravitational force: $n = F_g + F$.

PROBLEM-SOLVING HINTS

## Applying Newton's Laws

The following procedure is recommended when dealing with problems involving Newton's laws:

- Draw a simple, neat diagram of the system to help *conceptualize* the problem.
- *Categorize* the problem: if any acceleration component is zero, the particle is in equilibrium in this direction and $\Sigma F = 0$. If not, the particle is undergoing an acceleration, the problem is one of nonequilibrium in this direction, and $\Sigma F = ma$.
- *Analyze* the problem by isolating the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw *separate* free-body diagrams for each object. *Do not* include in the free-body diagram forces exerted by the object on its surroundings.
- Establish convenient coordinate axes for each object and find the components of the forces along these axes. Apply Newton's second law, $\Sigma \mathbf{F} = m\mathbf{a}$, in component form. Check your dimensions to make sure that all terms have units of force.
- Solve the component equations for the unknowns. Remember that you must have as many independent equations as you have unknowns to obtain a complete solution.
- *Finalize* by making sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

**Example 5.4 A Traffic Light at Rest**

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?

***Solution*** We *conceptualize* the problem by inspecting the drawing in Figure 5.10a. Let us assume that the cables do not break so that there is no acceleration of any sort in this problem in any direction. This allows us to *categorize* the problem as one of equilibrium. Because the acceleration of the system is zero, we know that the net force on the light and the net force on the knot are both zero. To *analyze* the

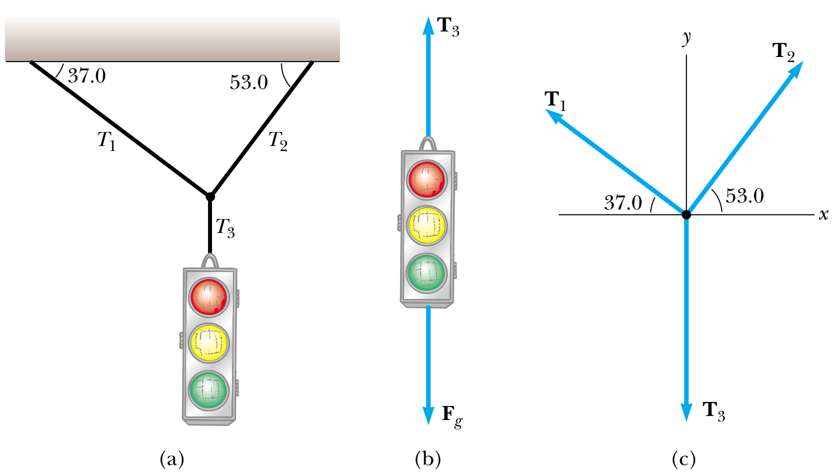



**Figure 5.10** (Example 5.4) (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot where the three cables are joined.

problem, we construct two free-body diagrams—one for the traffic light, shown in Figure 5.10b, and one for the knot that holds the three cables together, as in Figure 5.10c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.

With reference to Figure 5.10b, we apply the equilibrium condition in the $y$ direction, $\Sigma F_y = 0 \rightarrow T_3 - F_g = 0$. This leads to $T_3 = F_g = 122$ N. Thus, the upward force $\mathbf{T}_3$ exerted by the vertical cable on the light balances the gravitational force.

Next, we choose the coordinate axes shown in Figure 5.10c and resolve the forces acting on the knot into their components:

| Force | $x$ Component | $y$ Component |
|---|---|---|
| $\mathbf{T}_1$ | $-T_1 \cos 37.0°$ | $T_1 \sin 37.0°$ |
| $\mathbf{T}_2$ | $T_2 \cos 53.0°$ | $T_2 \sin 53.0°$ |
| $\mathbf{T}_3$ | 0 | $-122$ N |

Knowing that the knot is in equilibrium ($\mathbf{a} = 0$) allows us to write

$$(1) \qquad \sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$(2) \qquad \sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$$

From (1) we see that the horizontal components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must be equal in magnitude, and from (2) we see that the sum of the vertical components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must balance the downward force $\mathbf{T}_3$, which is equal in magnitude to the weight of the light. We solve (1) for $T_2$ in terms of $T_1$ to obtain

$$(3) \qquad T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$$

This value for $T_2$ is substituted into (2) to yield

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 122 \text{ N} = 0$$

$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33T_1 = 97.4 \text{ N}$$

Both of these values are less than 100 N (just barely for $T_2$), so the cables will not break. Let us *finalize* this problem by imagining a change in the system, as in the following **What If?**

**What If? Suppose the two angles in Figure 5.10a are equal. What would be the relationship between $T_1$ and $T_2$?**

***Answer*** We can argue from the symmetry of the problem that the two tensions $T_1$ and $T_2$ would be equal to each other. Mathematically, if the equal angles are called $\theta$, Equation (3) becomes

$$T_2 = T_1\left(\frac{\cos \theta}{\cos \theta}\right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of $\theta$, we cannot find the values of $T_1$ and $T_2$. However, the tensions will be equal to each other, regardless of the value of $\theta$.

## Conceptual Example 5.5 Forces Between Cars in a Train

Train cars are connected by *couplers*, which are under tension as the locomotive pulls the train. As you move through the train from the locomotive to the last car, does the tension in the couplers increase, decrease, or stay the same as the train speeds up? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from the locomotive to the last car? (Assume that only the brakes on the wheels of the engine are applied.)

***Solution*** As the train speeds up, tension decreases from front to back. The coupler between the locomotive and the first car must apply enough force to accelerate the rest of the cars. As you move back along the train, each coupler is accelerating less mass behind it. The last coupler has to accelerate only the last car, and so it is under the least tension.

When the brakes are applied, the force again decreases from front to back. The coupler connecting the locomotive to the first car must apply a large force to slow down the rest of the cars, but the final coupler must apply a force large enough to slow down *only* the last car.

## Example 5.6 The Runaway Car

A car of mass $m$ is on an icy driveway inclined at an angle $\theta$, as in Figure 5.11a.

**(A)** Find the acceleration of the car, assuming that the driveway is frictionless.

***Solution*** *Conceptualize* the situation using Figure 5.11a. From everyday experience, we know that a car on an icy incline will accelerate down the incline. (It will do the same thing as a car on a hill with its brakes not set.) This allows us to *categorize* the situation as a nonequilibrium problem—that is, one in which an object accelerates. Figure 5.11b shows the free-body diagram for the car, which we can use to *analyze* the problem. The only forces acting on the car are the normal force $\mathbf{n}$ exerted by the inclined plane, which acts perpendicular to the plane, and the gravitational force $\mathbf{F}_g = m\mathbf{g}$, which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with $x$ along the incline and $y$ perpendicular to it, as in Figure 5.11b. (It is possible to solve the problem with "standard" horizontal and vertical axes. You may want to try this, just for practice.) Then, we replace the gravitational force by a component of magnitude $mg \sin\theta$ along the positive $x$ axis and one of magnitude $mg\cos\theta$ along the negative $y$ axis.

Now we apply Newton's second law in component form, noting that $a_y = 0$:

$$(1) \qquad \sum F_x = mg \sin\theta = ma_x$$

$$(2) \qquad \sum F_y = n - mg \cos\theta = 0$$

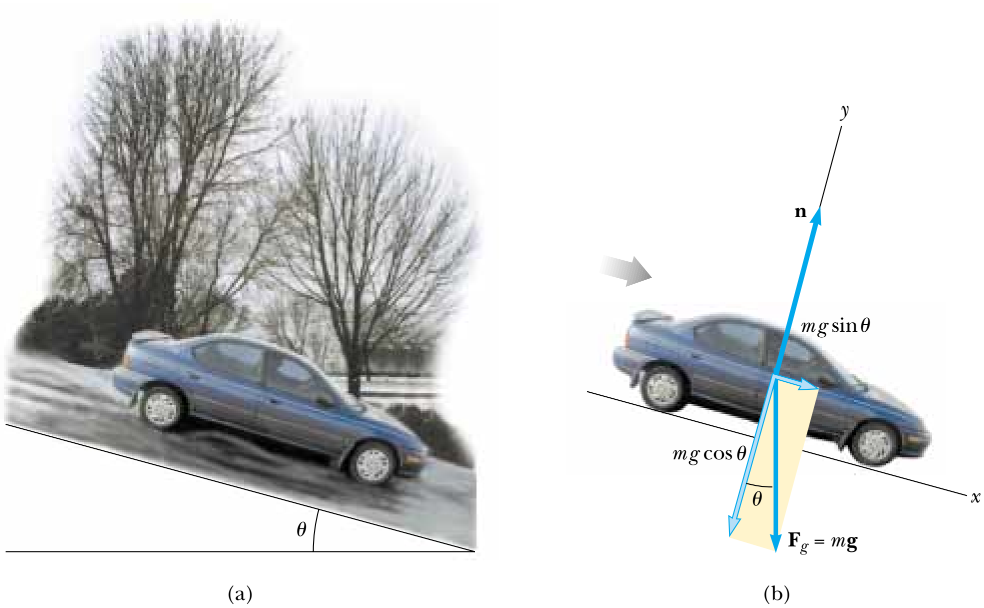



**Figure 5.11** (Example 5.6) (a) A car of mass $m$ sliding down a frictionless incline. (b) The free-body diagram for the car. Note that its acceleration along the incline is $g\sin\theta$.

Solving (1) for $a_x$, we see that the acceleration along the incline is caused by the component of $\mathbf{F}_g$ directed down the incline:

$$(3) \qquad a_x = \boxed{g \sin \theta}$$

To *finalize* this part, note that this acceleration component is independent of the mass of the car! It depends only on the angle of inclination and on $g$.

From (2) we conclude that the component of $\mathbf{F}_g$ perpendicular to the incline is balanced by the normal force; that is, $n = mg\cos\theta$. This is another example of a situation in which the normal force is *not* equal in magnitude to the weight of the object.

**(B)** Suppose the car is released from rest at the top of the incline, and the distance from the car's front bumper to the bottom of the incline is $d$. How long does it take the front bumper to reach the bottom, and what is the car's speed as it arrives there?

***Solution*** *Conceptualize* by imagining that the car is sliding down the hill and you are operating a stop watch to measure the entire time interval until it reaches the bottom. This part of the problem belongs to kinematics rather than to dynamics, and Equation (3) shows that the acceleration $a_x$ is constant. Therefore you should *categorize* this problem as that of a particle undergoing constant acceleration. Apply Equation 2.12, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, to *analyze* the car's motion. Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, we obtain

$$d = \tfrac{1}{2}a_x t^2$$

$$(4) \qquad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

Using Equation 2.13, with $v_{xi} = 0$, we find that

$$v_{xf}^{\,2} = 2a_x d$$

$$(5) \qquad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

To *finalize* this part of the problem, we see from Equations (4) and (5) that the time $t$ at which the car reaches the bottom and its final speed $v_{xf}$ are independent of the car's mass, as was its acceleration. Note that we have combined techniques from Chapter 2 with new techniques from the present chapter in this example. As we learn more and more techniques in later chapters, this process of combining information from several parts of the book will occur more often. In these cases, use the General Problem-Solving Strategy to help you identify what techniques you will need.

**What If? (A) What previously solved problem does this become if $\theta = 90°$? (B) What problem does this become if $\theta = 0$?**

***Answer*** (A) Imagine $\theta$ going to 90° in Figure 5.11. The inclined plane becomes vertical, and the car is an object in free-fall! Equation (3) becomes

$$a_x = g\sin\theta = g\sin 90° = g$$

which is indeed the free-fall acceleration. (We find $a_x = g$ rather than $a_x = -g$ because we have chosen positive $x$ to be downward in Figure 5.11.) Notice also that the condition

$n = mg\cos\theta$ gives us $n = mg\cos 90° = 0$. This is consistent with the fact that the car is falling downward *next to* the vertical plane but there is no interaction force between the car and the plane. Equations (4) and (5) give us $t = \sqrt{\dfrac{2d}{g\sin 90°}} = \sqrt{\dfrac{2d}{g}}$ and $v_{xf} = \sqrt{2gd\sin 90°} = \sqrt{2gd}$, both of which are consistent with a falling object.

(B) Imagine $\theta$ going to 0 in Figure 5.11. In this case, the inclined plane becomes horizontal, and the car is on a horizontal surface. Equation (3) becomes

$$a_x = g\sin\theta = g\sin 0 = 0$$

which is consistent with the fact that the car is at rest in equilibrium. Notice also that the condition $n = mg\cos\theta$ gives us $n = mg\cos 0 = mg$, which is consistent with our expectation.

Equations (4) and (5) give us $t = \sqrt{\dfrac{2d}{g\sin 0}} \rightarrow \infty$ and $v_{xf} = \sqrt{2gd\sin 0} = 0$. These results agree with the fact that the car does not accelerate, so it will never achieve a nonzero final velocity, and it will take an infinite amount of time to reach the bottom of the "hill"!

## Example 5.7 One Block Pushes Another

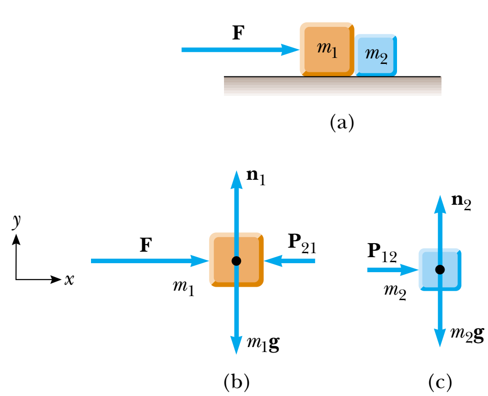
Two blocks of masses $m_1$ and $m_2$, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface, as in Figure 5.12a. A constant horizontal force $\mathbf{F}$ is applied to $m_1$ as shown. **(A)** Find the magnitude of the acceleration of the system.

***Solution*** *Conceptualize* the situation using Figure 5.12a and realizing that both blocks must experience the *same* acceleration because they are in contact with each other and remain in contact throughout the motion. We *categorize* this as a Newton's second law problem because we have a force applied to a system and we are looking for an acceleration. To *analyze* the problem, we first address the combination of two blocks as a system. Because $\mathbf{F}$ is the only external horizontal force acting on the system, we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

$$(1) \qquad a_x = \boxed{\frac{F}{m_1 + m_2}}$$

To *finalize* this part, note that this would be the same acceleration as that of a single object of mass equal to the combined masses of the two blocks in Figure 5.12a and subject to the same force.




**Active Figure 5.12** (Example 5.7) A force is applied to a block of mass $m_1$, which pushes on a second block of mass $m_2$. (b) The free-body diagram for $m_1$. (c) The free-body diagram for $m_2$.

***At the Active Figures link at* http://www.pse6.com, *you can study the forces involved in this two-block system.***

**(B)** Determine the magnitude of the contact force between the two blocks.

***Solution*** *Conceptualize* by noting that the contact force is internal to the system of two blocks. Thus, we cannot find this force by modeling the whole system (the two blocks) as a single particle. We must now treat each of the two blocks individually by *categorizing* each as a particle subject to a net force. To *analyze* the situation, we first construct a free-body diagram for each block, as shown in Figures 5.12b and 5.12c, where the contact force is denoted by $\mathbf{P}$. From Figure 5.12c we see that the only horizontal force acting on $m_2$ is the contact force $\mathbf{P}_{12}$ (the force exerted by $m_1$ on $m_2$), which is directed to the right. Applying Newton's second law to $m_2$ gives

$$(2) \qquad \sum F_x = P_{12} = m_2 a_x$$

Substituting the value of the acceleration $a_x$ given by (1) into (2) gives

$$(3) \qquad P_{12} = m_2 a_x = \boxed{\left(\frac{m_2}{m_1 + m_2}\right)F}$$

To *finalize* the problem, we see from this result that the contact force $P_{12}$ is *less* than the applied force $F$. This is consistent with the fact that the force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

To *finalize* further, it is instructive to check this expression for $P_{12}$ by considering the forces acting on $m_1$, shown in Figure 5.12b. The horizontal forces acting on $m_1$ are the applied force $\mathbf{F}$ to the right and the contact force $\mathbf{P}_{21}$ to the left (the force exerted by $m_2$ on $m_1$). From Newton's third law, $\mathbf{P}_{21}$ is the reaction to $\mathbf{P}_{12}$, so $P_{21} = P_{12}$. Applying Newton's second law to $m_1$ gives

$$(4) \qquad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

Substituting into (4) the value of $a_x$ from (1), we obtain

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

This agrees with (3), as it must.

**What If?** **Imagine that the force F in Figure 5.12 is applied toward the left on the right-hand block of mass $m_2$. Is the magnitude of the force $P_{12}$ the same as it was when the force was applied toward the right on $m_1$?**

***Answer*** With the force applied toward the left on $m_2$, the contact force must accelerate $m_1$. In the original situation, the contact force accelerates $m_2$. Because $m_1 > m_2$, this will require more force, so the magnitude of $\mathbf{P}_{12}$ is greater than in the original situation.

## Example 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass $m$ on a spring scale attached to the ceiling of an elevator, as illustrated in Figure 5.13. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

***Solution*** *Conceptualize* by noting that the reading on the scale is related to the extension of the spring in the scale, which is related to the force on the end of the spring as in Figure 5.2. Imagine that a string is hanging from the end of the spring, so that the magnitude of the force exerted on the spring is equal to the tension $T$ in the string. Thus, we are looking for $T$. The force $\mathbf{T}$ pulls down on the string and pulls up on the fish. Thus, we can *categorize* this problem as one of analyzing the forces and acceleration associated with the fish by means of Newton's second law. To *analyze* the problem, we inspect the free-body diagrams for the fish in Figure 5.13 and note that the external forces acting on the fish are the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the force $\mathbf{T}$ exerted by the scale. If the elevator is either at rest or moving at constant velocity, the fish does not accelerate, and so $\sum F_y = T - F_g = 0$ or $T = F_g = mg$. (Remember that the scalar $mg$ is the weight of the fish.)

If the elevator moves with an acceleration $\mathbf{a}$ relative to an observer standing outside the elevator in an inertial frame (see Fig. 5.13), Newton's second law applied to the fish gives the net force on the fish:

$$(1) \qquad \sum F_y = T - mg = ma_y$$

where we have chosen upward as the positive $y$ direction. Thus, we conclude from (1) that the scale reading $T$ is greater than the fish's weight $mg$ if $\mathbf{a}$ is upward, so that $a_y$ is positive, and that the reading is less than $mg$ if $\mathbf{a}$ is downward, so that $a_y$ is negative.

For example, if the weight of the fish is 40.0 N and $\mathbf{a}$ is upward, so that $a_y = +2.00 \text{ m/s}^2$, the scale reading from (1) is

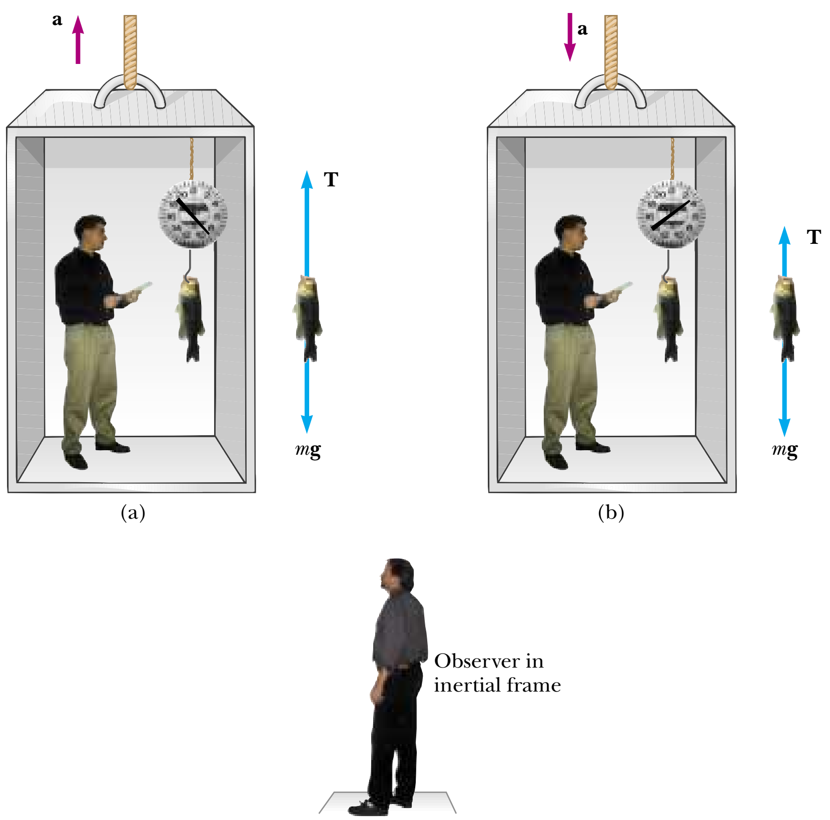



**Figure 5.13** (Example 5.8) Apparent weight versus true weight. (a) When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish. (b) When the elevator accelerates downward, the spring scale reads a value less than the weight of the fish.

$$(2)\qquad T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right)$$
$$= F_g\left(\frac{a_y}{g} + 1\right) = (40.0\text{ N})\left(\frac{2.00\text{ m/s}^2}{9.80\text{ m/s}^2} + 1\right)$$
$$= \boxed{48.2\text{ N}}$$

If **a** is downward so that $a_y = -2.00\text{ m/s}^2$, then (2) gives us

$$T = F_g\left(\frac{a_y}{g} + 1\right) = (40.0\text{ N})\left(\frac{-2.00\text{ m/s}^2}{9.80\text{ m/s}^2} + 1\right)$$
$$= \boxed{31.8\text{ N}}$$

To *finalize* this problem, take this advice—if you buy a fish in an elevator, make sure the fish is weighed while the elevator is either at rest or accelerating downward! Furthermore, note that from the information given here, one cannot determine the direction of motion of the elevator.

**What If? Suppose the elevator cable breaks, so that the elevator and its contents are in free-fall. What happens to the reading on the scale?**

***Answer*** If the elevator falls freely, its acceleration is $a_y = -g$. We see from (2) that the scale reading $T$ is zero in this case; that is, the fish *appears* to be weightless.

## Example 5.9 The Atwood Machine

Interactive

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in Figure 5.14a, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

***Solution*** *Conceptualize* the situation pictured in Figure 5.14a—as one object moves upward, the other object moves downward. Because the objects are connected by an inextensible string, their accelerations must be of equal magnitude. The objects in the Atwood machine are subject to the gravitational force as well as to the forces exerted by the strings connected to them—thus, we can *categorize* this as a Newton's second law problem. To *analyze* the situation, the free-body diagrams for the two objects are shown in Figure 5.14b. Two forces act on each object: the upward force **T** exerted by the string and the downward gravitational force. In problems such as this in which the pulley is modeled as massless and frictionless, the tension in the string on both sides of the pulley is the same. If the pulley has mass and/or is subject to friction, the tensions on either side are not the same and the situation requires techniques we will learn in Chapter 10.

We must be very careful with signs in problems such as this. In Figure 5.14a, notice that if object 1 accelerates upward, then object 2 accelerates downward. Thus, for consistency with signs, if we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2. With this sign convention, both objects accelerate in the same direction as defined by the choice of sign. Furthermore, according to this sign convention, the $y$ component of the net force exerted on object 1 is $T - m_1g$, and the $y$ component of the net force exerted on object 2 is $m_2g - T$. Notice that we have chosen the signs of the forces to be consistent with the choices of signs for up and down for each object. If we assume that $m_2 > m_1$, then $m_1$ must accelerate upward, while $m_2$ must accelerate downward.

When Newton's second law is applied to object 1, we obtain

$$(1)\qquad \sum F_y = T - m_1g = m_1a_y$$

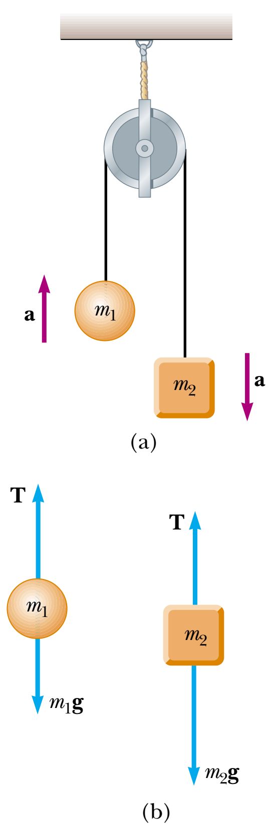
Similarly, for object 2 we find

$$(2)\qquad \sum F_y = m_2g - T = m_2a_y$$

When (2) is added to (1), $T$ cancels and we have

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

$$(3)\qquad a_y = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2}\right)g}$$




**Active Figure 5.14** (Example 5.9) The Atwood machine. (a) Two objects ($m_2 > m_1$) connected by a massless inextensible cord over a frictionless pulley. (b) Free-body diagrams for the two objects.

***At the Active Figures link at* http://www.pse6.com, *you can adjust the masses of the objects on the Atwood machine and observe the motion.***

The acceleration given by (3) can be interpreted as the ratio of the magnitude of the unbalanced force on the system $(m_2 - m_1)g$, to the total mass of the system $(m_1 + m_2)$, as expected from Newton's second law.

When (3) is substituted into (1), we obtain

$$(4) \qquad T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

*Finalize* this problem with the following **What If?**

**What If? (A) Describe the motion of the system if the objects have equal masses, that is, $m_1 = m_2$.**

**(B) Describe the motion of the system if one of the masses is much larger than the other, $m_1 \gg m_2$.**

***Answer*** (A) If we have the same mass on both sides, the system is balanced and it should not accelerate. Mathematically, we see that if $m_1 = m_2$, Equation (3) gives us $a_y = 0$.

(B) In the case in which one mass is infinitely larger than the other, we can ignore the effect of the smaller mass. Thus, the larger mass should simply fall as if the smaller mass were not there. We see that if $m_1 \gg m_2$, Equation (3) gives us $a_y = -g$.

*Investigate these limiting cases at the Interactive Worked Example link at* http://www.pse6.com.

## Example 5.10 Acceleration of Two Objects Connected by a Cord

**Interactive**

A ball of mass $m_1$ and a block of mass $m_2$ are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in Figure 5.15a. The block lies on a frictionless incline of angle $\theta$. Find the magnitude of the acceleration of the two objects and the tension in the cord.

***Solution*** *Conceptualize* the motion in Figure 5.15. If $m_2$ moves down the incline, $m_1$ moves upward. Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude. We can identify forces on each of the two objects and we are looking for an acceleration, so we *categorize* this as a Newton's second-law problem. To *analyze* the problem, consider the free-body diagrams shown in Figures 5.15b and 5.15c. Applying Newton's second law in component form to the ball, choosing the upward direction as positive, yields

$$(1) \qquad \sum F_x = 0$$

$$(2) \qquad \sum F_y = T - m_1g = m_1a_y = m_1a$$

Note that in order for the ball to accelerate upward, it is necessary that $T > m_1g$. In (2), we replaced $a_y$ with $a$ because the acceleration has only a $y$ component.

For the block it is convenient to choose the positive $x'$ axis along the incline, as in Figure 5.15c. For consistency with our choice for the ball, we choose the positive direction to be down the incline. Applying Newton's second law in component form to the block gives

$$(3) \qquad \sum F_{x'} = m_2g\sin\theta - T = m_2a_{x'} = m_2a$$

$$(4) \qquad \sum F_{y'} = n - m_2g\cos\theta = 0$$

In (3) we replaced $a_{x'}$ with $a$ because the two objects have accelerations of equal magnitude $a$. Equations (1) and (4) provide no information regarding the acceleration. However, if we solve (2) for $T$ and then substitute this value for $T$ into (3) and solve for $a$, we obtain

$$(5) \qquad a = \frac{m_2g\sin\theta - m_1g}{m_1 + m_2}$$

When this expression for $a$ is substituted into (2), we find

$$(6) \qquad T = \frac{m_1m_2g(\sin\theta + 1)}{m_1 + m_2}$$

To *finalize* the problem, note that the block accelerates down the incline only if $m_2\sin\theta > m_1$. If $m_1 > m_2\sin\theta$,

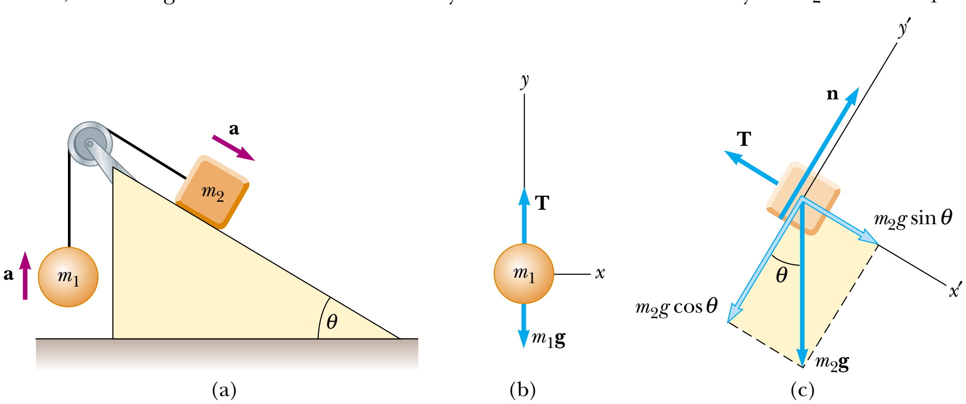



**Figure 5.15** (Example 5.10) (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) Free-body diagram for the ball. (c) Free-body diagram for the block. (The incline is frictionless.)

then the acceleration is up the incline for the block and downward for the ball. Also note that the result for the acceleration (5) can be interpreted as the magnitude of the net force acting on the system divided by the total mass of the system; this is consistent with Newton's second law.

**What If? (A) What happens in this situation if the angle $\theta = 90°$?**

**(B) What happens if the mass $m_1 = 0$?**

***Answer*** (A) If $\theta = 90°$, the inclined plane becomes vertical and there is no interaction between its surface and $m_2$. Thus, this problem becomes the Atwood machine of Example 5.9. Letting $\theta \rightarrow 90°$ in Equations (5) and (6) causes them to reduce to Equations (3) and (4) of Example 5.9! (B) If $m_1 = 0$, then $m_2$ is simply sliding down an inclined plane without interacting with $m_1$ through the string. Thus, this problem becomes the sliding car problem in Example 5.6. Letting $m_1 \rightarrow 0$ in Equation (5) causes it to reduce to Equation (3) of Example 5.6!

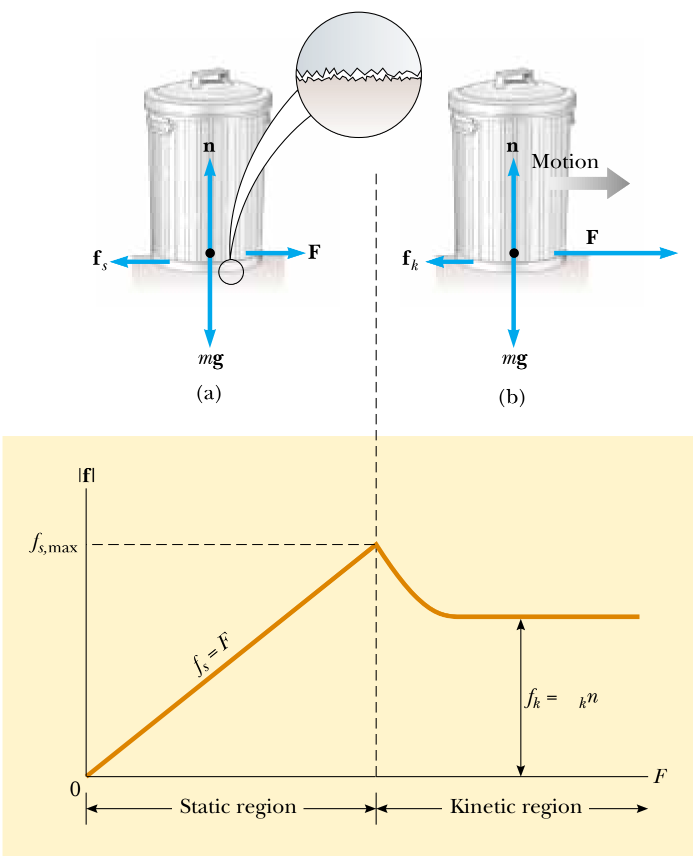
***Investigate these limiting cases at the Interactive Worked Example link at*** http://www.pse6.com.

## 5.8 Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction.** Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Imagine that you are working in your garden and have filled a trash can with yard clippings. You then try to drag the trash can across the surface of your concrete patio, as in Figure 5.16a. This is a *real* surface, not an idealized, frictionless surface. If we apply an external horizontal force **F** to the trash can, acting to the right, the trash can remains stationary if **F** is small. The force that counteracts **F** and keeps the trash can from moving acts to the left and is called the **force of static friction $\mathbf{f}_s$.** As long as the trash can is not moving, $f_s = F$. Thus, if **F** is increased, $\mathbf{f}_s$ also increases. Likewise, if **F** decreases, $\mathbf{f}_s$ also

**Force of static friction**




(c)

**Active Figure 5.16** The direction of the force of friction **f** between a trash can and a rough surface is opposite the direction of the applied force **F**. Because both surfaces are rough, contact is made only at a few points, as illustrated in the "magnified" view. (a) For small applied forces, the magnitude of the force of static friction equals the magnitude of the applied force. (b) When the magnitude of the applied force exceeds the magnitude of the maximum force of static friction, the trash can breaks free. The applied force is now larger than the force of kinetic friction and the trash can accelerates to the right. (c) A graph of friction force versus applied force. Note that $f_{s,\max} > f_k$.

***At the Active Figures link at*** **http://www.pse6.com** ***you can vary the applied force on the trash can and practice sliding it on surfaces of varying roughness. Note the effect on the trash can's motion and the corresponding behavior of the graph in (c).***

decreases. Experiments show that the friction force arises from the nature of the two surfaces: because of their roughness, contact is made only at a few locations where peaks of the material touch, as shown in the magnified view of the surface in Figure 5.16a.

At these locations, the friction force arises in part because one peak physically blocks the motion of a peak from the opposing surface, and in part from chemical bonding ("spot welds") of opposing peaks as they come into contact. If the surfaces are rough, bouncing is likely to occur, further complicating the analysis. Although the details of friction are quite complex at the atomic level, this force ultimately involves an electrical interaction between atoms or molecules.

If we increase the magnitude of **F**, as in Figure 5.16b, the trash can eventually slips. When the trash can is on the verge of slipping, $f_s$ has its maximum value $f_{s,\max}$, as shown in Figure 5.16c. When $F$ exceeds $f_{s,\max}$, the trash can moves and accelerates to the right. When the trash can is in motion, the friction force is less than $f_{s,\max}$ (Fig. 5.16c). We call the friction force for an object in motion the **force of kinetic friction** $\mathbf{f}_k$. The net force $F - f_k$ in the $x$ direction produces an acceleration to the right, according to Newton's second law. If $F = f_k$, the acceleration is zero, and the trash can moves to the right with constant speed. If the applied force is removed, the friction force acting to the left provides an acceleration of the trash can in the $-x$ direction and eventually brings it to rest, again consistent with Newton's second law.

Force of kinetic friction

Experimentally, we find that, to a good approximation, both $f_{s,\max}$ and $f_k$ are proportional to the magnitude of the normal force. The following empirical laws of friction summarize the experimental observations:

- The magnitude of the force of static friction between any two surfaces in contact can have the values

$$f_s \leq \mu_s n \tag{5.8}$$

where the dimensionless constant $\mu_s$ is called the **coefficient of static friction** and $n$ is the magnitude of the normal force exerted by one surface on the other. The equality in Equation 5.8 holds when the surfaces are on the verge of slipping, that is, when $f_s = f_{s,\max} \equiv \mu_s n$. This situation is called *impending motion.* The inequality holds when the surfaces are not on the verge of slipping.

- The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n \tag{5.9}$$

where $\mu_k$ is the **coefficient of kinetic friction.** Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in this text.

**PITFALL PREVENTION**

### 5.9 The Equal Sign is Used in Limited Situations

In Equation 5.8, the equal sign is used *only* in the case in which the surfaces are just about to break free and begin sliding. Do not fall into the common trap of using $f_s = \mu_s n$ in *any* static situation.

**PITFALL PREVENTION**

### 5.10 Friction Equations

Equations 5.8 and 5.9 are *not* vector equations. They are relationships between the *magnitudes* of the vectors representing the friction and normal forces. Because the friction and normal forces are perpendicular to each other, the vectors cannot be related by a multiplicative constant.

**Table 5.2**

**Coefficients of Friction**[a]

| | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Rubber on concrete | 1.0 | 0.8 |
| Wood on wood | 0.25–0.5 | 0.2 |
| Glass on glass | 0.94 | 0.4 |
| Waxed wood on wet snow | 0.14 | 0.1 |
| Waxed wood on dry snow | — | 0.04 |
| Metal on metal (lubricated) | 0.15 | 0.06 |
| Ice on ice | 0.1 | 0.03 |
| Teflon on Teflon | 0.04 | 0.04 |
| Synovial joints in humans | 0.01 | 0.003 |

[a] All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

- The values of $\mu_k$ and $\mu_s$ depend on the nature of the surfaces, but $\mu_k$ is generally less than $\mu_s$. Typical values range from around 0.03 to 1.0. Table 5.2 lists some reported values.
- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
- The coefficients of friction are nearly independent of the area of contact between the surfaces. We might expect that placing an object on the side having the most area might increase the friction force. While this provides more points in contact, as in Figure 5.16a, the weight of the object is spread out over a larger area, so that the individual points are not pressed so tightly together. These effects approximately compensate for each other, so that the friction force is independent of the area.

**PITFALL PREVENTION**

**5.11 The Direction of the Friction Force**

Sometimes, an incorrect statement about the friction force between an object and a surface is made—"the friction force on an object is opposite to its motion or impending motion"—rather than the correct phrasing, "the friction force on an object is opposite to its motion or impending motion *relative to the surface*." Think carefully about Quick Quiz 5.12.

**Quick Quiz 5.11** You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book? (a) downward (b) upward (c) out from the wall (d) into the wall.

**Quick Quiz 5.12** A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it, not sliding at all. What is the direction of the friction force exerted by the truck on the crate? (a) to the west (b) to the east (c) No friction force exists because the crate is not sliding.

**Quick Quiz 5.13** You place your physics book on a wooden board. You raise one end of the board so that the angle of the incline increases. Eventually, the book starts sliding on the board. If you maintain the angle of the board at this value, the book (a) moves at constant speed (b) speeds up (c) slows down (d) none of these.

**Quick Quiz 5.14** You are playing with your daughter in the snow. She sits on a sled and asks you to slide her across a flat, horizontal field. You have a choice of (a) pushing her from behind, by applying a force downward on her shoulders at 30° below the horizontal (Fig. 5.17a), or (b) attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal (Fig 5.17b). Which would be easier for you and why?


**Figure 5.17** (Quick Quiz 5.14) A father pushes his daughter on a sled either by (a) pushing down on her shoulders, or (b) pulling up on a rope.

## Conceptual Example 5.11 Why Does the Sled Accelerate?

A horse pulls a sled along a level, snow-covered road, causing the sled to accelerate, as shown in Figure 5.18a. Newton's third law states that the sled exerts a force of equal magnitude and opposite direction on the horse. In view of this, how can the sled accelerate—don't the forces cancel? Under what condition does the system (horse plus sled) move with constant velocity?

***Solution*** Remember that the forces described in Newton's third law act on *different* objects—the horse exerts a force on the sled, and the sled exerts an equal-magnitude and oppositely directed force on the horse. Because we are interested only in the motion of the sled, we do not consider the forces it exerts on the horse. When determining the motion of an object, you must add only the forces on that object. (This is the principle behind drawing a free-body diagram.) The horizontal forces exerted on the sled are the forward force $\mathbf{T}$ exerted by the horse and the backward force of friction $\mathbf{f}_{\text{sled}}$ between sled and snow (see Fig. 5.18b). When the forward force on the sled exceeds the backward force, the sled accelerates to the right.

The horizontal forces exerted on the horse are the forward force $\mathbf{f}_{\text{horse}}$ exerted by the Earth and the backward tension force $\mathbf{T}$ exerted by the sled (Fig. 5.18c). The resultant of these two forces causes the horse to accelerate.

The force that accelerates the system (horse plus sled) is the net force $\mathbf{f}_{\text{horse}} - \mathbf{f}_{\text{sled}}$. When $\mathbf{f}_{\text{horse}}$ balances $\mathbf{f}_{\text{sled}}$, the system moves with constant velocity.

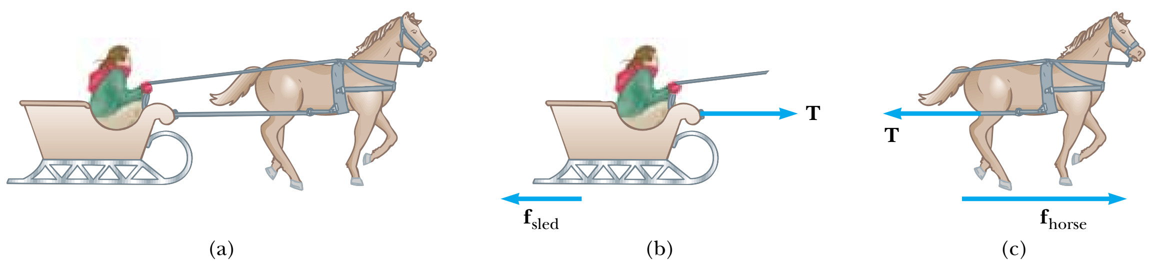



**Figure 5.18** (Conceptual Example 5.11)

## Example 5.12 Experimental Determination of $\mu_s$ and $\mu_k$

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure 5.19. The incline angle is increased until the block starts to move. Show that by measuring the critical angle $\theta_c$ at which this slipping just occurs, we can obtain $\mu_s$.

***Solution*** *Conceptualizing* from the free body diagram in Figure 5.19, we see that we can *categorize* this as a Newton's second law problem. To *analyze* the problem, note that the only forces acting on the block are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of static friction $\mathbf{f}_s$. These forces balance when the block is not moving. When we choose $x$ to be parallel to the plane and $y$ perpendicular to it, Newton's second law applied to the block for this balanced situation gives

$$(1) \qquad \sum F_x = mg \sin\theta - f_s = ma_x = 0$$

$$(2) \qquad \sum F_y = n - mg\cos\theta = ma_y = 0$$

We can eliminate $mg$ by substituting $mg = n/\cos\theta$ from (2) into (1) to find

$$(3) \qquad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s n$. The angle $\theta$ in this situation is the critical angle $\theta_c$, and (3) becomes

$$\mu_s n = n\tan\theta_c$$

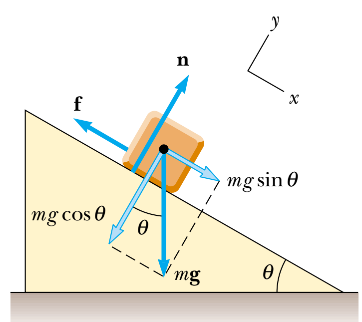



**Figure 5.19** (Example 5.12) The external forces exerted on a block lying on a rough incline are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of friction $\mathbf{f}$. For convenience, the gravitational force is resolved into a component along the incline $mg\sin\theta$ and a component perpendicular to the incline $mg\cos\theta$.

$$\mu_s = \tan\theta_c$$

For example, if the block just slips at $\theta_c = 20.0°$, then we find that $\mu_s = \tan 20.0° = 0.364$.

To *finalize* the problem, note that once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. However, if $\theta$ is reduced to a value less than $\theta_c$, it may be possible to find an angle $\theta_c'$ such that the block moves down the incline with constant speed ($a_x = 0$). In this case, using (1) and (2) with $f_s$ replaced by $f_k$ gives

$$\mu_k = \tan\theta_c'$$

where $\theta_c' < \theta_c$.

### Example 5.13 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

***Solution*** *Conceptualize* the problem by imagining that the puck in Figure 5.20 slides to the right and eventually comes to rest. To *categorize* the problem, note that we have forces identified in Figure 5.20, but that kinematic variables are provided in the text of the problem. Thus, we must combine the techniques of Chapter 2 with those of this chapter. (We assume that the friction force is constant, which will result in a constant horizontal acceleration.) To *analyze* the situation, note that the forces acting on the puck after it is in motion are shown in Figure 5.20. First, we find the acceleration algebraically in terms of the coefficient of kinetic friction, using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use the equations of kinematics to find the numerical value of the coefficient of kinetic friction.

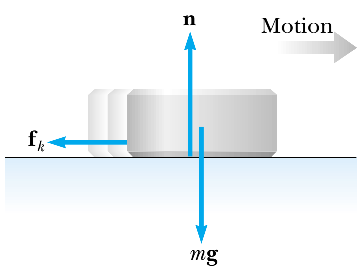



**Figure 5.20** (Example 5.13) *After* the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of kinetic friction $\mathbf{f}_k$.

Defining rightward and upward as our positive directions, we apply Newton's second law in component form to the puck and obtain

$$(1) \qquad \sum F_x = -f_k = ma_x$$

$$(2) \qquad \sum F_y = n - mg = 0 \qquad (a_y = 0)$$

But $f_k = \mu_k n$, and from (2) we see that $n = mg$. Therefore, (1) becomes

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left in Figure 5.20; because the velocity of the puck is to the right, this means that the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume that $\mu_k$ remains constant.

Because the acceleration is constant, we can use Equation 2.13, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_f = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(115\ \text{m})} = \boxed{0.117}$$

To *finalize* the problem, note that $\mu_k$ is dimensionless, as it should be, and that it has a low value, consistent with an object sliding on ice.

### Example 5.14 Acceleration of Two Connected Objects When Friction Is Present

A block of mass $m_1$ on a rough, horizontal surface is connected to a ball of mass $m_2$ by a lightweight cord over a lightweight, frictionless pulley, as shown in Figure 5.21a. A force of magnitude $F$ at an angle $\theta$ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is $\mu_k$. Determine the magnitude of the acceleration of the two objects.

***Solution*** *Conceptualize* the problem by imagining what happens as $\mathbf{F}$ is applied to the block. Assuming that $\mathbf{F}$ is not large enough to lift the block, the block will slide to the right and the ball will rise. We can identify forces and we want an acceleration, so we *categorize* this as a Newton's second law problem, one that includes the friction force. To *analyze* the problem, we begin by drawing free-body diagrams for the two objects, as shown in Figures 5.21b and 5.21c. Next, we apply Newton's second law in component form to each object and use Equation 5.9, $f_k = \mu_k n$. Then we can solve for the acceleration in terms of the parameters given.

The applied force $\mathbf{F}$ has $x$ and $y$ components $F\cos\theta$ and $F\sin\theta$, respectively. Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain

Motion of block: (1) $\quad \sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

(2) $\quad \sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$

Motion of ball: $\quad \sum F_x = m_2 a_x = 0$

(3) $\quad \sum F_y = T - m_2 g = m_2 a_y = m_2 a$

Because the two objects are connected, we can equate the magnitudes of the $x$ component of the acceleration of the block and the $y$ component of the acceleration of the ball. From Equation 5.9 we know that $f_k = \mu_k n$, and from (2) we know that $n = m_1 g - F\sin\theta$ (in this case $n$ is *not* equal to $m_1 g$); therefore,

$$(4) \qquad f_k = \mu_k(m_1 g - F\sin\theta)$$

That is, the friction force is reduced because of the positive $y$ component of $\mathbf{F}$. Substituting (4) and the value of $T$ from (3) into (1) gives

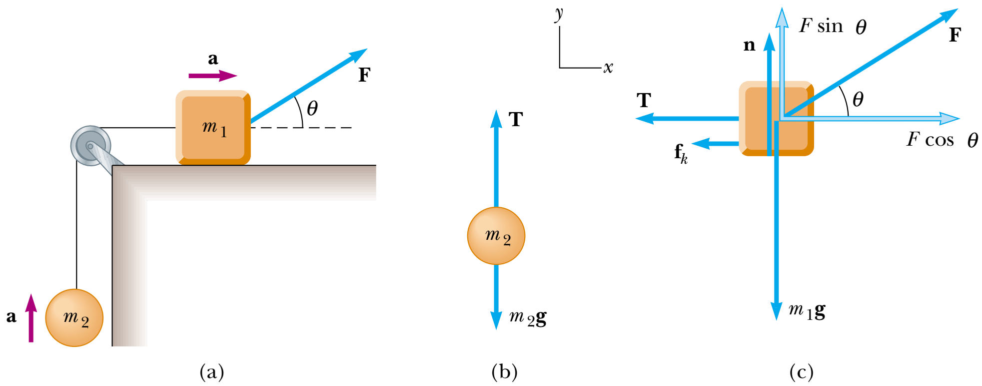



**Figure 5.21** (Example 5.14) (a) The external force **F** applied as shown can cause the block to accelerate to the right. (b) and (c) The free-body diagrams assuming that the block accelerates to the right and the ball accelerates upward. The magnitude of the force of kinetic friction in this case is given by $f_k = \mu_k n = \mu_k(m_1 g - F\sin\theta)$.

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a$$

Solving for $a$, we obtain

$$(5) \qquad a = \frac{F(\cos\theta + \mu_k \sin\theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}$$

To *finalize* the problem, note that the acceleration of the block can be either to the right or to the left,[5] depending on the sign of the numerator in (5). If the motion is to the left, then we must reverse the sign of $f_k$ in (1) because the force of kinetic friction must oppose the motion of the block relative to the surface. In this case, the value of $a$ is the same as in (5), with the two plus signs in the numerator changed to minus signs.

This is the final chapter in which we will explicitly show the steps of the General Problem-Solving Strategy in all worked examples. We will refer to them explicitly in occasional examples in future chapters, but you should use the steps in *all* of your problem solving.

[5] Equation 5 shows that when $\mu_k m_1 > m_2$, there is a range of values of $F$ for which no motion occurs at a given angle $\theta$.

## Application Automobile Antilock Braking Systems (ABS)

If an automobile tire is rolling and not slipping on a road surface, then the maximum friction force that the road can exert on the tire is the force of static friction $\mu_s n$. One must use static friction in this situation because at the point of contact between the tire and the road, no sliding of one surface over the other occurs if the tire is not skidding. However, if the tire starts to skid, the friction force exerted on it is reduced to the force of kinetic friction $\mu_k n$. Thus, to maximize the friction force and minimize stopping distance, the wheels must maintain pure rolling motion and not skid. An additional benefit of maintaining wheel rotation is that directional control is not lost as it is in skidding. Unfortunately, in emergency situations drivers typically press down as hard as they can on the brake pedal, "locking the brakes." This stops the wheels from rotating, ensuring a skid and reducing the friction force from the static to the kinetic value. To address this problem, automotive engineers have developed antilock braking systems (ABS). The purpose of the ABS is to help typical drivers (whose tendency is to lock the wheels in an emergency) to better maintain control of their automobiles and minimize stopping distance. The system briefly releases the brakes when a wheel is just about to stop turning. This maintains rolling contact between the tire and the pavement. When the brakes are released momentarily, the stopping distance is greater than it would be if the brakes were being applied continuously. However, through the use of computer control, the "brake-off" time is kept to a minimum. As a result, the stopping distance is much less than what it would be if the wheels were to skid.

Let us model the stopping of a car by examining real data. In an issue of *AutoWeek*,[6] the braking performance for a Toyota Corolla was measured. These data correspond to the braking force acquired by a highly trained, professional driver. We begin by assuming constant acceleration. (Why do we need to make this assumption?) The magazine provided the initial speed and stopping distance in non-SI units, which we show in the left and middle sections of Table 5.3. After converting these values to SI, we use $v_f^2 = v_i^2 + 2ax$ to determine the acceleration at different speeds, shown in the right section. These do not vary greatly, and so our assumption of constant acceleration is reasonable.

[6] *AutoWeek* magazine, **48:**22–23, 1998.

**Table 5.3**

**Data for a Toyota Corolla:**

| Initial Speed (mi/h) | Initial Speed (m/s) | Stopping Distance (ft) | Stopping Distance (m) | Acceleration ($m/s^2$) |
|---|---|---|---|---|
| 30 | 13.4 | 34 | 10.4 | −8.63 |
| 60 | 26.8 | 143 | 43.6 | −8.24 |
| 80 | 35.8 | 251 | 76.5 | −8.38 |

**Table 5.4**

**Stopping Distance With and Without Skidding**

| Initial Speed (mi/h) | Stopping Distance no skid (m) | Stopping Distance skidding (m) |
|---|---|---|
| 30 | 10.4 | 13.9 |
| 60 | 43.6 | 55.5 |
| 80 | 76.5 | 98.9 |

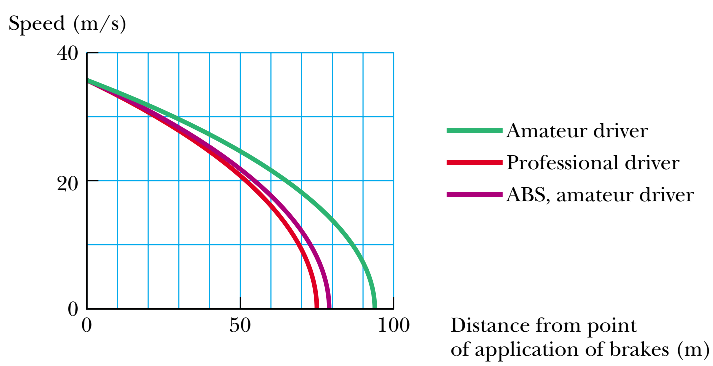



**Figure 5.22** This plot of vehicle speed versus distance from the location at which the brakes were applied shows that an antilock braking system (ABS) approaches the performance of a trained professional driver.

We take an average value of acceleration of $-8.4\ m/s^2$, which is approximately $0.86g$. We then calculate the coefficient of friction from $\Sigma F = \mu_s mg = ma$, which gives $\mu_s = 0.86$ for the Toyota. This is lower than the rubber-on-concrete value given in Table 5.2. Can you think of any reasons for this?

We now estimate the stopping distance of the car if the wheels were skidding. From Table 5.2, we see that the difference between the coefficients of static and kinetic friction for rubber against concrete is about 0.2. Let us assume that our coefficients differ by the same amount, so that $\mu_k \approx 0.66$. This allows us to estimate the stopping distances when the wheels are locked and the car skids across the pavement, as shown in the third column of Table 5.4. The results illustrate the advantage of not allowing the wheels to skid.

Because an ABS keeps the wheels rotating, the higher coefficient of static friction is maintained between the tires and road. This approximates the technique of a professional driver who is able to maintain the wheels at the point of maximum friction force. Let us estimate the ABS performance by assuming that the magnitude of the acceleration is not quite as good as that achieved by the professional driver but instead is reduced by 5%.

Figure 5.22 is a plot of vehicle speed versus distance from where the brakes were applied (at an initial speed of 80.0 mi/h = 35.8 m/s) for the three cases of amateur driver, professional driver, and estimated ABS performance (amateur driver). This shows that a markedly shorter distance is necessary for stopping without locking the wheels compared to skidding. In addition a satisfactory value of stopping distance is achieved when the ABS computer maintains tire rotation.

## SUMMARY

*Take a practice test for this chapter by clicking the Practice Test link at http://www.pse6.com.*

An **inertial frame of reference** is one we can identify in which an object that does not interact with other objects experiences zero acceleration. Any frame moving with constant velocity relative to an inertial frame is also an inertial frame. **Newton's first law** states that it is possible to find such a frame, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

**Newton's second law** states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force acting on an object equals the product of its mass and its acceleration: $\Sigma\mathbf{F} = m\mathbf{a}$. If the object is either stationary or moving with constant velocity, then the object is in equilibrium and the force vectors must cancel each other.

The **gravitational force** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration: $\mathbf{F}_g = m\mathbf{g}$. The **weight** of an object is the magnitude of the gravitational force acting on the object.

**Newton's third law** states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1. Thus, an isolated force cannot exist in nature.

The **maximum force of static friction** $\mathbf{f}_{s,\text{max}}$ between an object and a surface is proportional to the normal force acting on the object. In general, $f_s \le \mu_s n$, where $\mu_s$ is the **coefficient of static friction** and $n$ is the magnitude of the normal force. When an object slides over a surface, the direction of the **force of kinetic friction** $\mathbf{f}_k$ is opposite the direction of motion of the object relative to the surface and is also proportional to the magnitude of the normal force. The magnitude of this force is given by $f_k = \mu_k n$, where $\mu_k$ is the **coefficient of kinetic friction.**

## QUESTIONS

**1.** A ball is held in a person's hand. (a) Identify all the external forces acting on the ball and the reaction to each. (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Neglect air resistance.)

**2.** If a car is traveling westward with a constant speed of 20 m/s, what is the resultant force acting on it?

**3.** What is wrong with the statement "Because the car is at rest, there are no forces acting on it"? How would you correct this sentence?

**$\boxed{4.}$** In the motion picture *It Happened One Night* (Columbia Pictures, 1934), Clark Gable is standing inside a stationary bus in front of Claudette Colbert, who is seated. The bus suddenly starts moving forward and Clark falls into Claudette's lap. Why did this happen?

**5.** A passenger sitting in the rear of a bus claims that she was injured as the driver slammed on the brakes, causing a suitcase to come flying toward her from the front of the bus. If you were the judge in this case, what disposition would you make? Why?

**6.** A space explorer is moving through space far from any planet or star. She notices a large rock, taken as a specimen from an alien planet, floating around the cabin of the ship. Should she push it gently or kick it toward the storage compartment? Why?

**$\boxed{7.}$** A rubber ball is dropped onto the floor. What force causes the ball to bounce?

**8.** While a football is in flight, what forces act on it? What are the action–reaction pairs while the football is being kicked and while it is in flight?

**9.** The mayor of a city decides to fire some city employees because they will not remove the obvious sags from the cables that support the city traffic lights. If you were a lawyer, what defense would you give on behalf of the employees? Who do you think would win the case in court?

**$\boxed{10.}$** A weightlifter stands on a bathroom scale. He pumps a barbell up and down. What happens to the reading on the bathroom scale as this is done? **What if** he is strong enough to actually *throw* the barbell upward? How does the reading on the scale vary now?

**11.** Suppose a truck loaded with sand accelerates along a highway. If the driving force on the truck remains constant, what happens to the truck's acceleration if its trailer leaks sand at a constant rate through a hole in its bottom?

**12.** As a rocket is fired from a launching pad, its speed and acceleration increase with time as its engines continue to operate. Explain why this occurs even though the thrust of the engines remains constant.

**13.** What force causes an automobile to move? A propeller-driven airplane? A rowboat?

**14.** Identify the action–reaction pairs in the following situations: a man takes a step; a snowball hits a girl in the back; a baseball player catches a ball; a gust of wind strikes a window.

**15.** In a contest of National Football League behemoths, teams from the Rams and the 49ers engage in a tug-of-war, pulling in opposite directions on a strong rope. The Rams exert a force of 9 200 N and they are winning, making the center of the rope move steadily toward themselves. Is it possible to know the tension in the rope from the information stated? Is it larger or smaller than 9 200 N? How hard are the 49ers pulling on the rope? Would it change your answer if the 49ers were winning or if the contest were even? The stronger team wins by exerting a larger force—on what? Explain your answers.

**16.** Twenty people participate in a tug-of-war. The two teams of ten people are so evenly matched that neither team wins. After the game they notice that a car is stuck in the mud. They attach the tug-of-war rope to the bumper of the car, and all the people pull on the rope. The heavy car has just moved a couple of decimeters when the rope breaks. Why did the rope break in this situation when it did not break when the same twenty people pulled on it in a tug-of-war?

**17.** "When the locomotive in Figure Q5.17 broke through the wall of the train station, the force exerted by the locomotive on the wall was greater than the force the wall could exert on the locomotive." Is this statement true or in need of correction? Explain your answer.

**18.** An athlete grips a light rope that passes over a low-friction pulley attached to the ceiling of a gym. A sack of sand precisely equal in weight to the athlete is tied to the other end of the rope. Both the sand and the athlete are initially at rest. The athlete climbs the rope, sometimes speeding up and slowing down as he does so. What happens to the sack of sand? Explain.

**19.** If the action and reaction forces are always equal in magnitude and opposite in direction to each other, then doesn't the net vector force on any object necessarily add up to zero? Explain your answer.

**20.** Can an object exert a force on itself? Argue for your answer.

**21.** If you push on a heavy box that is at rest, you must exert some force to start its motion. However, once the box is sliding, you can apply a smaller force to maintain that motion. Why?

Roger Viollet, Mill Valley, CA, University Science Books, 1982

**Figure Q5.17**

**22.** The driver of a speeding empty truck slams on the brakes and skids to a stop through a distance $d$. (a) If the truck carried a load that doubled its mass, what would be the truck's "skidding distance"? (b) If the initial speed of the truck were halved, what would be the truck's skidding distance?

**23.** Suppose you are driving a classic car. Why should you avoid slamming on your brakes when you want to stop in the shortest possible distance? (Many cars have antilock brakes that avoid this problem.)

**24.** A book is given a brief push to make it slide up a rough incline. It comes to a stop and slides back down to the starting point. Does it take the same time to go up as to come down? **What if** the incline is frictionless?

**25.** A large crate is placed on the bed of a truck but not tied down. (a) As the truck accelerates forward, the crate remains at rest relative to the truck. What force causes the crate to accelerate forward? (b) If the driver slammed on the brakes, what could happen to the crate?

**26.** Describe a few examples in which the force of friction exerted on an object is in the direction of motion of the object.

# PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging  □ = full solution available in the *Student Solutions Manual and Study Guide*
= coached solution with hints available at http://www.pse6.com  = computer useful in solving problem
= paired numerical and symbolic problems

## Sections 5.1 through 5.6

1. A force **F** applied to an object of mass $m_1$ produces an acceleration of 3.00 m/s$^2$. The same force applied to a second object of mass $m_2$ produces an acceleration of 1.00 m/s$^2$. (a) What is the value of the ratio $m_1/m_2$? (b) If $m_1$ and $m_2$ are combined, find their acceleration under the action of the force **F**.

2. The largest-caliber antiaircraft gun operated by the German air force during World War II was the 12.8-cm Flak 40. This weapon fired a 25.8-kg shell with a muzzle speed of 880 m/s. What propulsive force was necessary to attain the muzzle speed within the 6.00-m barrel? (Assume the shell moves horizontally with constant acceleration and neglect friction.)

3. A 3.00-kg object undergoes an acceleration given by $\mathbf{a} = (2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ m/s$^2$. Find the resultant force acting on it and the magnitude of the resultant force.

4. The gravitational force on a baseball is $-F_g\hat{\mathbf{j}}$. A pitcher throws the baseball with velocity $v\hat{\mathbf{i}}$ by uniformly accelerating it straight forward horizontally for a time interval $\Delta t = t - 0 = t$. If the ball starts from rest, (a) through what distance does it accelerate before its release? (b) What force does the pitcher exert on the ball?

5. To model a spacecraft, a toy rocket engine is securely fastened to a large puck, which can glide with negligible friction over a horizontal surface, taken as the $xy$ plane. The 4.00-kg puck has a velocity of $300\hat{\mathbf{i}}$ m/s at one instant. Eight seconds later, its velocity is to be $(800\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}})$ m/s. Assuming the rocket engine exerts a constant horizontal force, find (a) the components of the force and (b) its magnitude.

6. The average speed of a nitrogen molecule in air is about $6.70 \times 10^2$ m/s, and its mass is $4.68 \times 10^{-26}$ kg. (a) If it takes $3.00 \times 10^{-13}$ s for a nitrogen molecule to hit a wall and rebound with the same speed but moving in the opposite direction, what is the average acceleration of the molecule during this time interval? (b) What average force does the molecule exert on the wall?

7. An electron of mass $9.11 \times 10^{-31}$ kg has an initial speed of $3.00 \times 10^5$ m/s. It travels in a straight line, and its speed increases to $7.00 \times 10^5$ m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the force exerted on the electron and (b) compare this force with the weight of the electron, which we neglected.

8. A woman weighs 120 lb. Determine (a) her weight in newtons (N) and (b) her mass in kilograms (kg).

9. If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s$^2$?

10. The distinction between mass and weight was discovered after Jean Richer transported pendulum clocks from Paris to French Guyana in 1671. He found that they ran slower there quite systematically. The effect was reversed when the clocks returned to Paris. How much weight would you personally lose in traveling from Paris, where $g$ = 9.809 5 m/s$^2$, to Cayenne, where $g$ = 9.780 8 m/s$^2$? [We will consider how the free-fall acceleration influences the period of a pendulum in Section 15.5.]

11. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a 5.00-kg object. If $F_1$ = 20.0 N and $F_2$ = 15.0 N, find the accelerations in (a) and (b) of Figure P5.11.

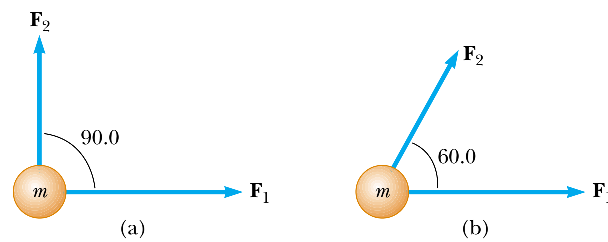


**Figure P5.11**

12. Besides its weight, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.20\hat{\mathbf{i}} - 3.30\hat{\mathbf{j}})$ m, where the direction of $\hat{\mathbf{j}}$ is the upward vertical direction. Determine the other force.

13. You stand on the seat of a chair and then hop off. (a) During the time you are in flight down to the floor, the Earth is lurching up toward you with an acceleration of what order of magnitude? In your solution explain your logic. Model the Earth as a perfectly solid object. (b) The Earth moves up through a distance of what order of magnitude?

14. Three forces acting on an object are given by $\mathbf{F}_1 = (-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}})$ N, $\mathbf{F}_2 = (5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ N, and $\mathbf{F}_3 = (-45.0\hat{\mathbf{i}})$ N. The object experiences an acceleration of magnitude 3.75 m/s$^2$. (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?

15. A 15.0-lb block rests on the floor. (a) What force does the floor exert on the block? (b) If a rope is tied to the block and run vertically over a pulley, and the other end is attached to a free-hanging 10.0-lb weight, what is the force exerted by the floor on the 15.0-lb block? (c) If we replace the 10.0-lb weight in part (b) with a 20.0-lb weight, what is the force exerted by the floor on the 15.0-lb block?

## Section 5.7 Some Applications of Newton's Laws

**16.** A 3.00-kg object is moving in a plane, with its $x$ and $y$ coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where $x$ and $y$ are in meters and $t$ is in seconds. Find the magnitude of the net force acting on this object at $t = 2.00$ s.

**17.** The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. Draw a free-body diagram of the bird. How much tension does the bird produce in the wire? Ignore the weight of the wire.

**18.** A bag of cement of weight 325 N hangs from three wires as suggested in Figure P5.18. Two of the wires make angles $\theta_1 = 60.0°$ and $\theta_2 = 25.0°$ with the horizontal. If the system is in equilibrium, find the tensions $T_1$, $T_2$, and $T_3$ in the wires.

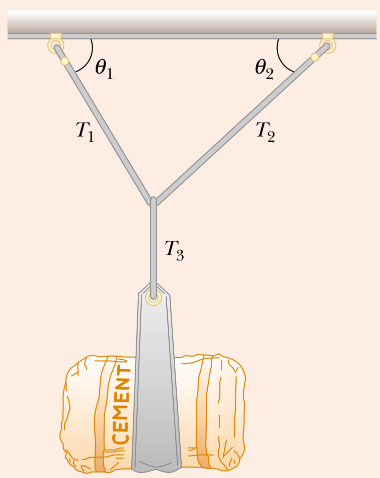



**Figure P5.18** Problems 18 and 19.

**19.** A bag of cement of weight $F_g$ hangs from three wires as shown in Figure P5.18. Two of the wires make angles $\theta_1$ and $\theta_2$ with the horizontal. If the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = F_g \cos\theta_2 / \sin(\theta_1 + \theta_2)$$

**20.** You are a judge in a children's kite-flying contest, and two children will win prizes for the kites that pull most strongly and least strongly on their strings. To measure string tensions, you borrow a weight hanger, some slotted weights, and a protractor from your physics teacher, and use the following protocol, illustrated in Figure P5.20: Wait for a child to get her kite well controlled, hook the hanger onto the kite string about 30 cm from her hand, pile on weight until that section of string is horizontal, record the mass required, and record the angle between the horizontal and the string running up to the kite. (a) Explain how this method works. As you construct your explanation, imagine that the children's parents ask you about your method, that they might make false assumptions about your ability without concrete evidence, and that your explanation is an opportunity to give them confidence in your evaluation technique. (b) Find the string tension if the mass is 132 g and the angle of the kite string is 46.3°.

**Figure P5.20**

**21.** The systems shown in Figure P5.21 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline in part (c) is frictionless.)

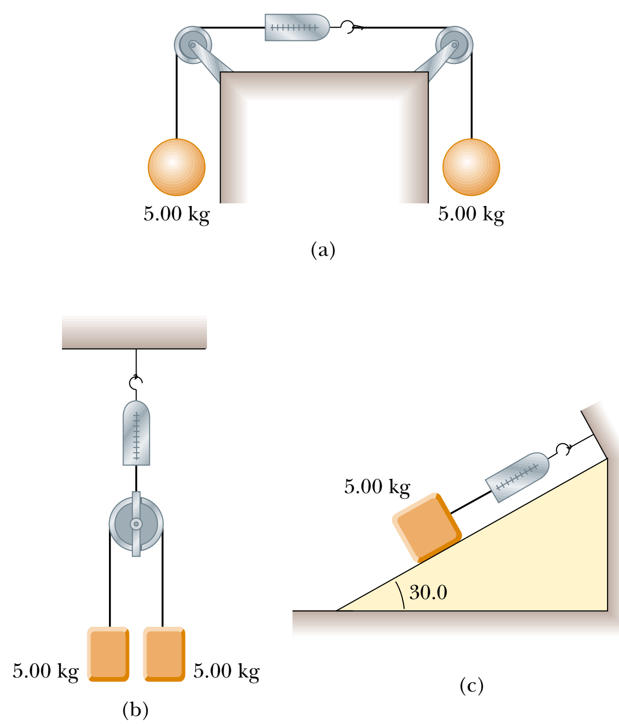



**Figure P5.21**

**22.** Draw a free-body diagram of a block which slides down a frictionless plane having an inclination of $\theta = 15.0°$ (Fig. P5.22). The block starts from rest at the top and the length of the incline is 2.00 m. Find (a) the acceleration of the block and (b) its speed when it reaches the bottom of the incline.

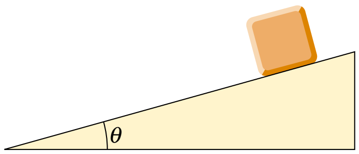



**Figure P5.22** Problems 22 and 25.

23. A 1.00-kg object is observed to have an acceleration of 10.0 $m/s^2$ in a direction 30.0° north of east (Fig. P5.23). The force $\mathbf{F}_2$ acting on the object has a magnitude of 5.00 N and is directed north. Determine the magnitude and direction of the force $\mathbf{F}_1$ acting on the object.

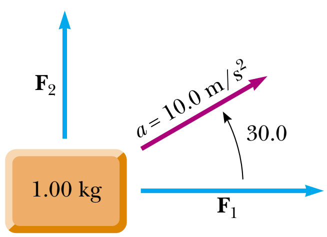



**Figure P5.23**

24. A 5.00-kg object placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 9.00-kg object, as in Figure P5.24. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string.

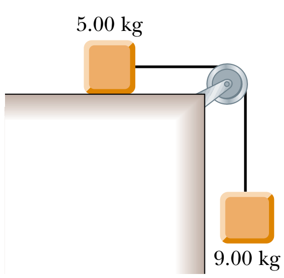



**Figure P5.24** Problems 24 and 43.

25. A block is given an initial velocity of 5.00 m/s up a frictionless 20.0° incline (Fig. P5.22). How far up the incline does the block slide before coming to rest?

26. Two objects are connected by a light string that passes over a frictionless pulley, as in Figure P5.26. Draw free-body diagrams of both objects. If the incline is frictionless and if $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$, find (a) the accelerations of the objects, (b) the tension in the string, and (c) the speed of each object 2.00 s after being released from rest.

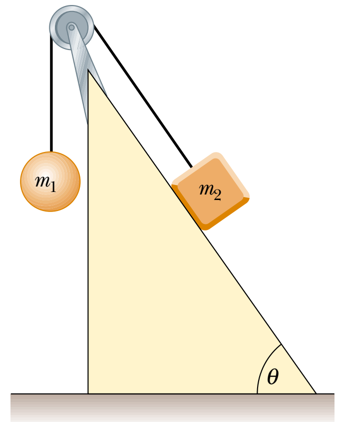



**Figure P5.26**

27. A tow truck pulls a car that is stuck in the mud, with a force of 2 500 N as in Fig. P5.27. The tow cable is under tension and therefore pulls downward and to the left on the pin at its upper end. The light pin is held in equilibrium by forces exerted by the two bars A and B. Each bar is a *strut*: that is, each is a bar whose weight is small compared to the forces it exerts, and which exerts forces only through hinge pins at its ends. Each strut exerts a force directed parallel to its length. Determine the force of tension or compression in each strut. Proceed as follows: Make a guess as to which way (pushing or pulling) each force acts on the top pin. Draw a free-body diagram of the pin. Use the condition for equilibrium of the pin to translate the free-body diagram into equations. From the equations calculate the forces exerted by struts A and B. If you obtain a positive answer, you correctly guessed the direction of the force. A negative answer means the direction should be reversed, but the absolute value correctly gives the magnitude of the force. If a strut pulls on a pin, it is in tension. If it pushes, the strut is in compression. Identify whether each strut is in tension or in compression.

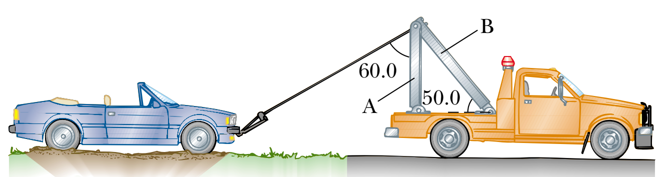



**Figure P5.27**

28. Two objects with masses of 3.00 kg and 5.00 kg are connected by a light string that passes over a light frictionless pulley to form an Atwood machine, as in Figure 5.14a. Determine (a) the tension in the string, (b) the acceleration of each object, and (c) the distance each object will move in the first second of motion if they start from rest.

29. In Figure P5.29, the man and the platform together weigh 950 N. The pulley can be modeled as frictionless. Determine how hard the man has to pull on the rope to lift himself steadily upward above the ground. (Or is it impossible? If so, explain why.)

**Figure P5.29**

**30.** In the Atwood machine shown in Figure 5.14a, $m_1 = 2.00$ kg and $m_2 = 7.00$ kg. The masses of the pulley and string are negligible by comparison. The pulley turns without friction and the string does not stretch. The lighter object is released with a sharp push that sets it into motion at $v_i = 2.40$ m/s downward. (a) How far will $m_1$ descend below its initial level? (b) Find the velocity of $m_1$ after 1.80 seconds.

**31.** In the system shown in Figure P5.31, a horizontal force $\mathbf{F}_x$ acts on the 8.00-kg object. The horizontal surface is frictionless. (a) For what values of $F_x$ does the 2.00-kg object accelerate upward? (b) For what values of $F_x$ is the tension in the cord zero? (c) Plot the acceleration of the 8.00-kg object versus $F_x$. Include values of $F_x$ from $-100$ N to $+100$ N.

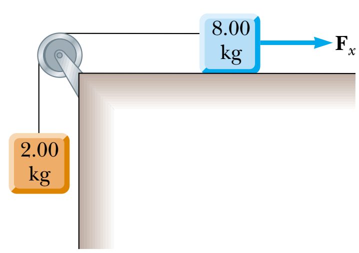



**Figure P5.31**

**32.** A frictionless plane is 10.0 m long and inclined at 35.0°. A sled starts at the bottom with an initial speed of 5.00 m/s up the incline. When it reaches the point at which it momentarily stops, a second sled is released from the top of this incline with an initial speed $v_i$. Both sleds reach the bottom of the incline at the same moment. (a) Determine the distance that the first sled traveled up the incline. (b) Determine the initial speed of the second sled.

**33.** A 72.0-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.20 m/s in 0.800 s. It travels with this constant speed for the next 5.00 s. The elevator then undergoes a uniform acceleration in the negative $y$ direction for 1.50 s and comes to rest. What does the spring scale register (a) before the elevator starts to move? (b) during the first 0.800 s? (c) while the elevator is traveling at constant speed? (d) during the time it is slowing down?

**34.** An object of mass $m_1$ on a frictionless horizontal table is connected to an object of mass $m_2$ through a very light pulley $P_1$ and a light fixed pulley $P_2$ as shown in Figure P5.34. (a) If $a_1$ and $a_2$ are the accelerations of $m_1$ and $m_2$, respectively, what is the relation between these accelerations? Express (b) the tensions in the strings and (c) the accelerations $a_1$ and $a_2$ in terms of the masses $m_1$ and $m_2$, and $g$.

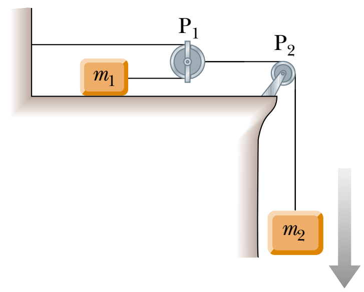



**Figure P5.34**

## Section 5.8 Forces of Friction

**35.** The person in Figure P5.35 weighs 170 lb. As seen from the front, each light crutch makes an angle of 22.0° with the vertical. Half of the person's weight is supported by the crutches. The other half is supported by the vertical forces of the ground on his feet. Assuming the person is moving

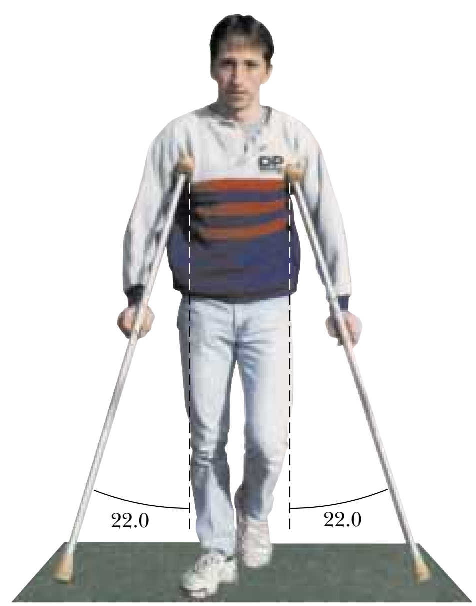



**Figure P5.35**

with constant velocity and the force exerted by the ground on the crutches acts along the crutches, determine (a) the smallest possible coefficient of friction between crutches and ground and (b) the magnitude of the compression force in each crutch.

**36.** A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

**37.** A car is traveling at 50.0 mi/h on a horizontal highway. (a) If the coefficient of static friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and $\mu_s = 0.600$?

**38.** Before 1960 it was believed that the maximum attainable coefficient of static friction for an automobile tire was less than 1. Then, about 1962, three companies independently developed racing tires with coefficients of 1.6. Since then, tires have improved, as illustrated in this problem. According to the 1990 Guinness Book of Records, the shortest time in which a piston-engine car initially at rest has covered a distance of one-quarter mile is 4.96 s. This record was set by Shirley Muldowney in September 1989. (a) Assume that, as in Figure P5.38, the rear wheels lifted the front wheels off the pavement. What minimum value of $\mu_s$ is necessary to achieve the record time? (b) Suppose Muldowney were able to double her engine power, keeping other things equal. How would this change affect the elapsed time?


Mike Powell/Allsport USA/Getty

**Figure P5.38**

**39.** To meet a U.S. Postal Service requirement, footwear must have a coefficient of static friction of 0.5 or more on a specified tile surface. A typical athletic shoe has a coefficient of 0.800. In an emergency, what is the minimum time interval in which a person starting from rest can move 3.00 m on a tile surface if she is wearing (a) footwear meeting the Postal Service minimum? (b) a typical athletic shoe?

**40.** A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle $\theta$ above the horizontal (Fig. P5.40). She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. Draw a free-body diagram of the suitcase. (a) What angle does the strap make with the horizontal? (b) What normal force does the ground exert on the suitcase?

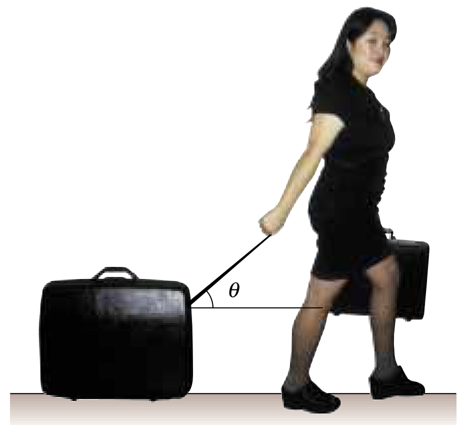



**Figure P5.40**

**41.** A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2.00 m.

**42.** A Chevrolet Corvette convertible can brake to a stop from a speed of 60.0 mi/h in a distance of 123 ft on a level roadway. What is its stopping distance on a roadway sloping downward at an angle of 10.0°?

**43.** A 9.00-kg hanging weight is connected by a string over a pulley to a 5.00-kg block that is sliding on a flat table (Fig. P5.24). If the coefficient of kinetic friction is 0.200, find the tension in the string.

**44.** Three objects are connected on the table as shown in Figure P5.44. The table is rough and has a coefficient of kinetic friction of 0.350. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. (a) Determine the acceleration of each object and their directions. (b) Determine the tensions in the two cords.

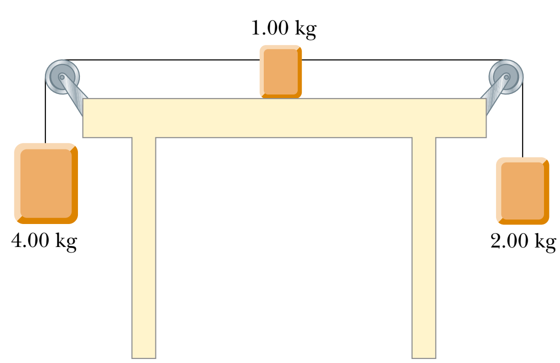



**Figure P5.44**

**45.** Two blocks connected by a rope of negligible mass are being dragged by a horizontal force **F** (Fig. P5.45). Suppose that $F = 68.0$ N, $m_1 = 12.0$ kg, $m_2 = 18.0$ kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block.

(b) Determine the tension $T$ and the magnitude of the acceleration of the system.

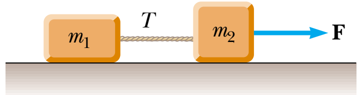



**Figure P5.45**

46. A block of mass 3.00 kg is pushed up against a wall by a force **P** that makes a 50.0° angle with the horizontal as shown in Figure P5.46. The coefficient of static friction between the block and the wall is 0.250. Determine the possible values for the magnitude of **P** that allow the block to remain stationary.

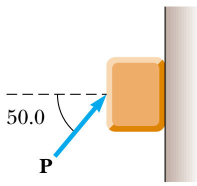



**Figure P5.46**

47. You and your friend go sledding. Out of curiosity, you measure the constant angle $\theta$ that the snow-covered slope makes with the horizontal. Next, you use the following method to determine the coefficient of friction $\mu_k$ between the snow and the sled. You give the sled a quick push up so that it will slide up the slope away from you. You wait for it to slide back down, timing the motion. It turns out that the sled takes twice as long to slide down as it does to reach the top point in the round trip. In terms of $\theta$, what is the coefficient of friction?

48. The board sandwiched between two other boards in Figure P5.48 weighs 95.5 N. If the coefficient of friction between the boards is 0.663, what must be the magnitude of the compression forces (assume horizontal) acting on both sides of the center board to keep it from slipping?

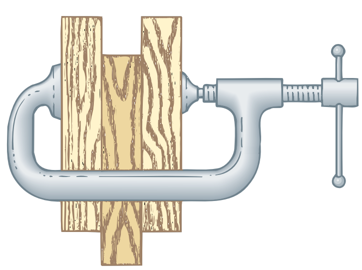


**Figure P5.48**

49. A block weighing 75.0 N rests on a plane inclined at 25.0° to the horizontal. A force $F$ is applied to the object at 40.0° to the horizontal, pushing it upward on the plane. The coefficients of static and kinetic friction between the block and the plane are, respectively, 0.363 and 0.156. (a) What is the minimum value of $F$ that will prevent the block from slipping down the plane? (b) What is the minimum value of $F$ that will start the block moving up the plane? (c) What value of $F$ will move the block up the plane with constant velocity?

50. **Review problem.** One side of the roof of a building slopes up at 37.0°. A student throws a Frisbee onto the roof. It strikes with a speed of 15.0 m/s and does not bounce, but slides straight up the incline. The coefficient of kinetic friction between the plastic and the roof is 0.400. The Frisbee slides 10.0 m up the roof to its peak, where it goes into free fall, following a parabolic trajectory with negligible air resistance. Determine the maximum height the Frisbee reaches above the point where it struck the roof.

## Additional Problems

51. An inventive child named Pat wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig. P5.51), Pat pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Pat's true weight is 320 N, and the chair weighs 160 N. (a) Draw free-body diagrams for Pat and the chair considered as separate systems, and another diagram for Pat and the chair considered as one system. (b) Show that the acceleration of the system is *upward* and find its magnitude. (c) Find the force Pat exerts on the chair.

**Figure P5.51**

52. A time-dependent force, $\mathbf{F} = (8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}})$ N, where $t$ is in seconds, is exerted on a 2.00-kg object initially at rest. (a) At what time will the object be moving with a speed of 15.0 m/s? (b) How far is the object from its initial position when its speed is 15.0 m/s? (c) Through what total displacement has the object traveled at this time?

53. To prevent a box from sliding down an inclined plane, student A pushes on the box in the direction parallel to the incline, just hard enough to hold the box stationary. In an identical situation student B pushes on the box horizontally. Regard as known the mass $m$ of the box, the coefficient of static friction $\mu_s$ between box and incline, and the inclination angle $\theta$. (a) Determine the force A

has to exert. (b) Determine the force B has to exert. (c) If $m = 2.00$ kg, $\theta = 25.0°$, and $\mu_s = 0.160$, who has the easier job? (d) **What if** $\mu_s = 0.380$? Whose job is easier?

**54.** Three blocks are in contact with each other on a frictionless, horizontal surface, as in Figure P5.54. A horizontal force **F** is applied to $m_1$. Take $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N. Draw a separate free-body diagram for each block and find (a) the acceleration of the blocks, (b) the *resultant* force on each block, and (c) the magnitudes of the contact forces between the blocks. (d) You are working on a construction project. A coworker is nailing up plasterboard on one side of a light partition, and you are on the opposite side, providing "backing" by leaning against the wall with your back pushing on it. Every blow makes your back sting. The supervisor helps you to put a heavy block of wood between the wall and your back. Using the situation analyzed in parts (a), (b), and (c) as a model, explain how this works to make your job more comfortable.

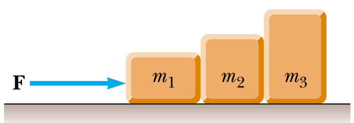


**Figure P5.54**

**55.** An object of mass $M$ is held in place by an applied force **F** and a pulley system as shown in Figure P5.55. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, $T_1$, $T_2$, $T_3$, $T_4$, and $T_5$ and (b) the magnitude of **F**. *Suggestion:* Draw a free-body diagram for each pulley.

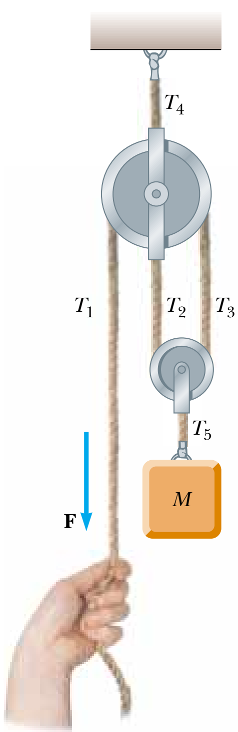


**Figure P5.55**

**56.** A high diver of mass 70.0 kg jumps off a board 10.0 m above the water. If his downward motion is stopped 2.00 s after he enters the water, what average upward force did the water exert on him?

**57.** A crate of weight $F_g$ is pushed by a force **P** on a horizontal floor. (a) If the coefficient of static friction is $\mu_s$ and **P** is directed at angle $\theta$ below the horizontal, show that the minimum value of $P$ that will move the crate is given by

$$P = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}$$

(b) Find the minimum value of $P$ that can produce motion when $\mu_s = 0.400$, $F_g = 100$ N, and $\theta = 0°$, 15.0°, 30.0°, 45.0°, and 60.0°.

**58. Review problem.** A block of mass $m = 2.00$ kg is released from rest at $h = 0.500$ m above the surface of a table, at the top of a $\theta = 30.0°$ incline as shown in Figure P5.58. The frictionless incline is fixed on a table of height $H = 2.00$ m. (a) Determine the acceleration of the block as it slides down the incline. (b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) How much time has elapsed between when the block is released and when it hits the floor? (e) Does the mass of the block affect any of the above calculations?

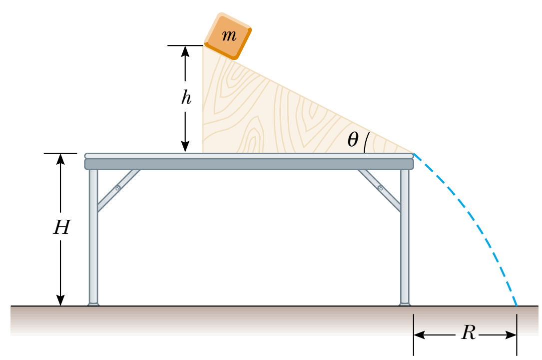


**Figure P5.58** Problems 58 and 70.

**59.** A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. (a) For the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) With this angle, how large must the tension be?

**60.** Materials such as automobile tire rubber and shoe soles are tested for coefficients of static friction with an apparatus called a James tester. The pair of surfaces for which $\mu_s$ is to be measured are labeled B and C in Figure P5.60. Sample C is attached to a foot D at the lower end of a pivoting arm E, which makes angle $\theta$ with the vertical. The upper end of the arm is hinged at F to a vertical rod G, which slides freely in a guide H fixed to the frame of the apparatus and supports a load I of mass 36.4 kg. The hinge pin at F is also the axle of a wheel that can roll vertically on the frame. All of the moving parts have masses negligible in comparison to the 36.4-kg load. The pivots are nearly frictionless. The test surface B is attached to a rolling platform A. The operator slowly moves the platform to the left in the picture until the sample C suddenly slips over surface B. At the critical point where sliding motion is ready to begin, the operator notes the angle $\theta_s$ of the pivoting arm. (a) Make a free-body diagram of the pin at F. It is in equilibrium under three forces. These forces are the gravitational force on the load I, a horizontal normal force exerted by the frame, and a force of compression directed upward along the arm E. (b) Draw a free-body diagram of the foot D and sample C, considered as one system. (c) Determine the normal force that the test surface B exerts on the sample for any angle $\theta$. (d) Show that $\mu_s = \tan \theta_s$. (e) The protractor on the tester can record angles as large as 50.2°. What is the greatest coefficient of friction it can measure?

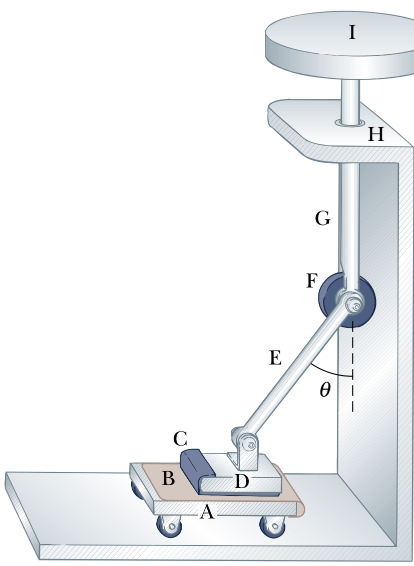



**Figure P5.60**

**61.** What horizontal force must be applied to the cart shown in Figure P5.61 in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (*Hint:* Note that the force exerted by the string accelerates $m_1$.)

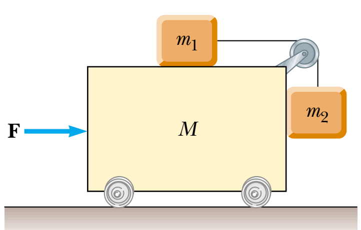



**Figure P5.61** Problems 61 and 63.

**62.** A student is asked to measure the acceleration of a cart on a "frictionless" inclined plane as in Figure 5.11, using an air track, a stopwatch, and a meter stick. The height of the incline is measured to be 1.774 cm, and the total length of the incline is measured to be $d = 127.1$ cm. Hence, the angle of inclination $\theta$ is determined from the relation $\sin \theta = 1.774/127.1$. The cart is released from rest at the top of the incline, and its position $x$ along the incline is measured as a function of time, where $x = 0$ refers to the initial position of the cart. For $x$ values of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm, 75.0 cm, and 100 cm, the measured times at which these positions are reached (averaged over five runs) are 1.02 s, 1.53 s, 2.01 s, 2.64 s, 3.30 s, and 3.75 s, respectively. Construct a graph of $x$ versus $t^2$, and perform a linear least-squares fit to the data. Determine the acceleration of the cart from the slope of this graph, and compare it with the value you would get using $a' = g \sin \theta$, where $g = 9.80 \text{ m/s}^2$.

**63.** Initially the system of objects shown in Figure P5.61 is held motionless. All surfaces, pulley, and wheels are frictionless. Let the force **F** be zero and assume that $m_2$ can move only vertically. At the instant after the system of objects is released, find (a) the tension $T$ in the string, (b) the acceleration of $m_2$, (c) the acceleration of $M$, and (d) the acceleration of $m_1$. (*Note:* The pulley accelerates along with the cart.)

**64.** One block of mass 5.00 kg sits on top of a second rectangular block of mass 15.0 kg, which in turn is on a horizontal table. The coefficients of friction between the two blocks are $\mu_s = 0.300$ and $\mu_k = 0.100$. The coefficients of friction between the lower block and the rough table are $\mu_s = 0.500$ and $\mu_k = 0.400$. You apply a constant horizontal force to the lower block, just large enough to make this block start sliding out from between the upper block and the table. (a) Draw a free-body diagram of each block, naming the forces on each. (b) Determine the magnitude of each force on each block at the instant when you have started pushing but motion has not yet started. In particular, what force must you apply? (c) Determine the acceleration you measure for each block.

**65.** A 1.00-kg glider on a horizontal air track is pulled by a string at an angle $\theta$. The taut string runs over a pulley and is attached to a hanging object of mass 0.500 kg as in Fig. P5.65. (a) Show that the speed $v_x$ of the glider and the

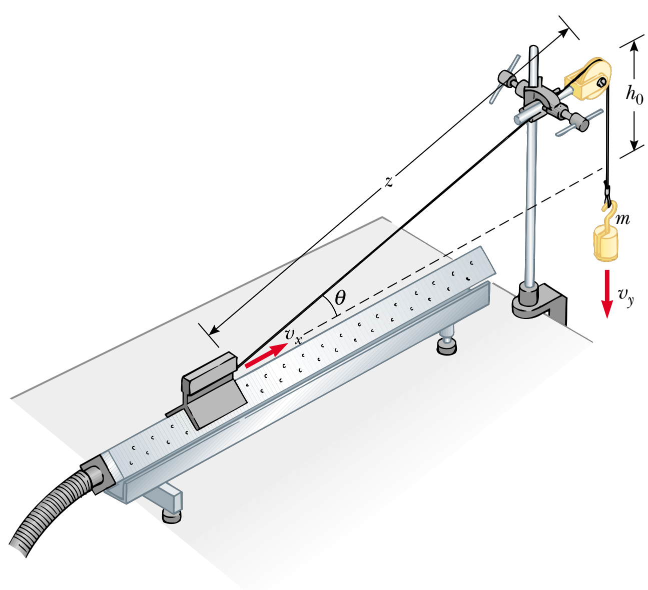



**Figure P5.65**

speed $v_y$ of the hanging object are related by $v_x = uv_y$, where $u = z(z^2 - h_0^2)^{-1/2}$. (b) The glider is released from rest. Show that at that instant the acceleration $a_x$ of the glider and the acceleration $a_y$ of the hanging object are related by $a_x = ua_y$. (c) Find the tension in the string at the instant the glider is released for $h_0 = 80.0$ cm and $\theta = 30.0°$.

**66.** Cam mechanisms are used in many machines. For example, cams open and close the valves in your car engine to admit gasoline vapor to each cylinder and to allow the escape of exhaust. The principle is illustrated in Figure P5.66, showing a follower rod (also called a pushrod) of mass $m$ resting on a wedge of mass $M$. The sliding wedge duplicates the function of a rotating eccentric disk on a camshaft in your car. Assume that there is no friction between the wedge and the base, between the pushrod and the wedge, or between the rod and the guide through which it slides. When the wedge is pushed to the left by the force $F$, the rod moves upward and does something, such as opening a valve. By varying the shape of the wedge, the motion of the follower rod could be made quite complex, but assume that the wedge makes a constant angle of $\theta = 15.0°$. Suppose you want the wedge and the rod to start from rest and move with constant acceleration, with the rod moving upward 1.00 mm in 8.00 ms. Take $m = 0.250$ kg and $M = 0.500$ kg. What force $F$ must be applied to the wedge?

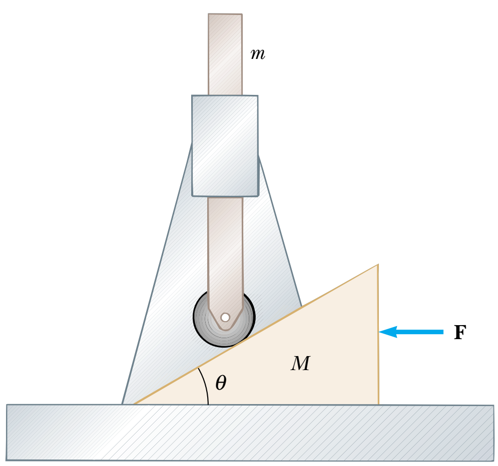



**Figure P5.66**

**67.** Any device that allows you to increase the force you exert is a kind of *machine*. Some machines, such as the prybar or the inclined plane, are very simple. Some machines do not even look like machines. An example is the following: Your car is stuck in the mud, and you can't pull hard enough to get it out. However, you have a long cable which you connect taut between your front bumper and the trunk of a stout tree. You now pull sideways on the cable at its midpoint, exerting a force $f$. Each half of the cable is displaced through a small angle $\theta$ from the straight line between the ends of the cable. (a) Deduce an expression for the force exerted on the car. (b) Evaluate the cable tension for the case where $\theta = 7.00°$ and $f = 100$ N.

**68.** Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley (Fig. P5.68). The inclines are frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.

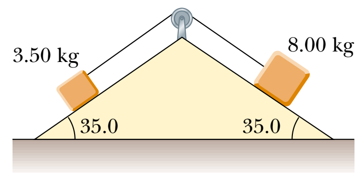



**Figure P5.68**

**69.** A van accelerates down a hill (Fig. P5.69), going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy ($m$ = 0.100 kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle $\theta$ and (b) the tension in the string.

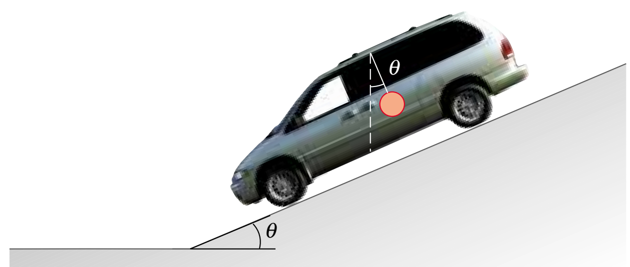



**Figure P5.69**

**70.** In Figure P5.58 the incline has mass $M$ and is fastened to the stationary horizontal tabletop. The block of mass $m$ is placed near the bottom of the incline and is released with a quick push that sets it sliding upward. It stops near the top of the incline, as shown in the figure, and then slides down again, always without friction. Find the force that the tabletop exerts on the incline throughout this motion.

**71.** A magician pulls a tablecloth from under a 200-g mug located 30.0 cm from the edge of the cloth. The cloth exerts a friction force of 0.100 N on the mug, and the cloth is pulled with a constant acceleration of 3.00 m/s$^2$. How far does the mug move relative to the horizontal tabletop before the cloth is completely out from under it? Note that the cloth must move more than 30 cm relative to the tabletop during the process.

**72.** An 8.40-kg object slides down a fixed, frictionless inclined plane. Use a computer to determine and tabulate the normal force exerted on the object and its acceleration for a series of incline angles (measured from the horizontal) ranging from 0° to 90° in 5° increments. Plot a graph of the normal force and the acceleration as functions of the incline angle. In the limiting cases of 0° and 90°, are your results consistent with the known behavior?

73. A mobile is formed by supporting four metal butterflies of equal mass $m$ from a string of length $L$. The points of support are evenly spaced a distance $\ell$ apart as shown in Figure P5.73. The string forms an angle $\theta_1$ with the ceiling at each end point. The center section of string is horizontal. (a) Find the tension in each section of string in terms of $\theta_1$, $m$, and $g$. (b) Find the angle $\theta_2$, in terms of $\theta_1$, that the sections of string between the outside butterflies and the inside butterflies form with the horizontal. (c) Show that the distance $D$ between the end points of the string is

$$D = \frac{L}{5}\left(2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\tfrac{1}{2}\tan\theta_1\right)\right] + 1\right)$$

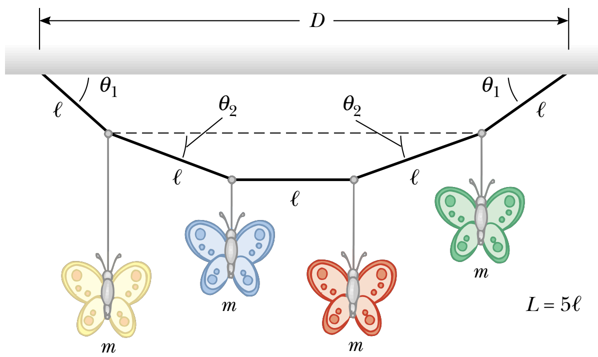



**Figure P5.73**

## Answers to Quick Quizzes

**5.1** (d). Choice (a) is true. Newton's first law tells us that motion requires no force: an object in motion continues to move at constant velocity in the absence of external forces. Choice (b) is also true. A stationary object can have several forces acting on it, but if the vector sum of all these external forces is zero, there is no net force and the object remains stationary.

**5.2** (a). If a single force acts, this force constitutes the net force and there is an acceleration according to Newton's second law.

**5.3** (c). Newton's second law relates only the force and the acceleration. Direction of motion is part of an object's *velocity,* and force determines the direction of acceleration, not that of velocity.

**5.4** (d). With twice the force, the object will experience twice the acceleration. Because the force is constant, the acceleration is constant, and the speed of the object (starting from rest) is given by $v = at$. With twice the acceleration, the object will arrive at speed $v$ at half the time.

**5.5** (a). The gravitational force acts on the ball at *all* points in its trajectory.

**5.6** (b). Because the value of $g$ is smaller on the Moon than on the Earth, more mass of gold would be required to represent 1 newton of weight on the Moon. Thus, your friend on the Moon is richer, by about a factor of 6!

**5.7** (c). In accordance with Newton's third law, the fly and bus experience forces that are equal in magnitude but opposite in direction.

**5.8** (a). Because the fly has such a small mass, Newton's second law tells us that it undergoes a very large acceleration. The huge mass of the bus means that it more effectively resists any change in its motion and exhibits a small acceleration.

**5.9** (c). The reaction force to your weight is an upward gravitational force on the Earth due to you.

**5.10** (b). Remember the phrase "free-body." You draw *one* body (one object), free of all the others that may be interacting, and draw only the forces exerted on that object.

**5.11** (b). The friction force acts opposite to the gravitational force on the book to keep the book in equilibrium. Because the gravitational force is downward, the friction force must be upward.

**5.12** (b). The crate accelerates to the east. Because the only horizontal force acting on it is the force of static friction between its bottom surface and the truck bed, that force must also be directed to the east.

**5.13** (b). At the angle at which the book breaks free, the component of the gravitational force parallel to the board is approximately equal to the maximum static friction force. Because the kinetic coefficient of friction is smaller than the static coefficient, at this angle, the component of the gravitational force parallel to the board is larger than the kinetic friction force. Thus, there is a net downhill force parallel to the board and the book speeds up.

**5.14** (b). When pulling with the rope, there is a component of your applied force that is upward. This reduces the normal force between the sled and the snow. In turn, this reduces the friction force between the sled and the snow, making it easier to move. If you push from behind, with a force with a downward component, the normal force is larger, the friction force is larger, and the sled is harder to move.